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DYNAMIC FEEDBACK STABILIZATION OF NONLINEAR RC LADDER NETWORK

Summary. The goal of this paper is to study stabilization techniques for a class of RC ladder networks. The system is an electrical circuit that consists of nonlinear resistors and capacitors. The circuit's dynamic behavior can be modeled by nonlinear differential equations. The problem is to determine the dynamic feedback control law that asymptotically stabilizes the system. It is shown that with a linear and nonlinear dynamic feedback control, the energy of the closed-loop system asymptotically decays to zero. The asymptotic stability of the closed-loop system is proved by LaSalle's invariance principle using appropriate Lyapunov function. The results of numerical computations are included to verify theoretical analysis and mathematical formulation.

Keywords: nonlinear RC ladder network, stabilization, dynamic feedback, Lyapunov function

STABILIZACJA NIELINIOWYCH OBWODÓW DRABINKOWYCH RC ZA POMOCĄ DYNAMICZNEGO SPRZĘŻENIA ZWROTNEGO

Streszczenie. W pracy rozważono zagadnienie stabilizacji dla wybranej klasy układów drabinkowych typu RC. Wybrana klasa układów obejmuje obwody elektryczne, składające się z rezystorów i kondensatorów o nieliniowych charakterystykach. Dynamika układu jest opisywana za pomocą nieliniowych równań różniczkowych. W pracy pokazano, że zastosowanie dynamicznego sprzężenia zwrotnego asymptotycznie stabilizuje system do zera. Rozważono zarówno liniowe, jak i nieliniowe sprzężenie zwrotne. Własność asymptotycznej stabilności układu zamkniętego została pokazana z wykorzystaniem odpowiednich funkcjonałów Lapunowa oraz twierdzenia LaSalle'a. Wyniki teoretyczne zostały zweryfikowane przez obliczenia numeryczne i symulacje komputerowe.

Słowa kluczowe: nieliniowe obwody drabinkowe RC, stabilizacja, dynamiczne sprzężenie zwrotne, funkcjonały Lapunowa

1. INTRODUCTION

Almost all real systems are nonlinear and it is well known that nonlinearity requires advanced analysis [1, 2, 5, 14, 23]. The dynamics of nonlinear system is difficult to analyze

and introduces to interesting phenomena such as bifurcations, limit cycles and chaos. On the other hand, nonlinear electronic elements have a wide range of use in many areas of electrical engineering. They are incorporated into circuits in order to design electronic devices with specific features like parametric amplifiers, up-converters, mixers, low-power microwave oscillators, electronic tuning devices, etc. Typical nonlinear elements include nonlinear capacitors (varactor diode, junction diode), nonlinear inductors (saturable core inductor, Josephson junctions, ferroresonant power systems) and nonlinear resistors (tunnel diode, thyristor, dead-zone conductor, serially connected Zener diodes, neon bulb, etc.).

The properties of electrical ladder networks have been a subject of earlier research. Control problems for linear RL, RC, LC and RLC electrical circuits are widely discussed in [7-9]. The dynamics and detailed characteristics of nonlinear electrical circuits are considered in [6]. The papers [21, 22] cope with linear and nonlinear stabilization techniques for a nonlinear RLC circuit. To stabilize the system, Authors have constructed various forms of the feedback. The asymptotic stability of the closed-loop system has been proved by LaSalle's invariance principle [4] using special Lyapunov functions. In [20] stabilization problem of nonlinear RLC ladder networks with linear dynamic feedback control was considered. Research included in this paper was mainly motivated and inspired by results obtained in [3, 10-13, 15-19]. They have played a crucial role and cleared the way to the main results.

The paper is organized as follows. In the next two sections, we shortly describe nonlinear resistors and nonlinear capacitors. In Section 4, a nonlinear RC ladder network is introduced and its dynamics is described. In Section 5, linear dynamic feedback control law is proposed and it is proved that this feedback asymptotically stabilizes the system. Section 6 is devoted to the system stabilization using nonlinear dynamic feedback control. In Section 7, a simulation example is presented. Concluding remarks are given in Section 8.

2. NONLINEAR RESISTORS

A resistor is an electric device that defines static relation between voltage and current. This static relation is represented by the equation $r(v,i)=0$, where v and i denote the voltage and the current and $r:\Re\times\Re\to\Re$ is a function.

A resistor is linear if the function r is linear. In that case, $r(v,i)$ can take several forms from which the most famous is impendence form: $v=Ri$, where R is the resistance measured in Ohms [Ω]. This formula is also known as Ohms' law.

A resistor is nonlinear, if r is nonlinear in the relation $r(v,i)$ that links the voltage v to the current i. We call a resistor dissipative if for all real numbers v and i we have that

$$r(v,i)=0 \quad \Rightarrow \quad vi\geq 0. \tag{1}$$

The product vi represents the power supplied to the component; therefore a dissipative resistor is characterized by the property that no voltage-current pair can produce negative power. Most resistors in electronics are dissipative components but non-dissipative resistors certainly exist and are nowadays assembled using semiconductors.

In this paper, it is assumed that the voltage drops across resistors presented in Fig. 1 can be modeled as

$$v_{R_k}(t) = R(i_k) i_k(t) = R(\dot{p}_k) \frac{\mathrm{d}p_k}{\mathrm{d}t}, \tag{2}$$

where i_k denote the currents in the circuit, p_k represent the electric charges, R_k stand for the resistances, $k = 1,2,\ldots,n$.

3. NONLINEAR CAPACITORS

A capacitor is defined as an electric component whose charge is a function of voltage. Its capacitance is defined as the derivative of the charge q with respect to the voltage v

$$C(q) = \frac{\mathrm{d}q(v)}{\mathrm{d}v}. \tag{3}$$

The current flowing through a capacitor is simply the time derivative of the charge

$$i(t) = \frac{\mathrm{d}q(v(t))}{\mathrm{d}t}. \tag{4}$$

If a capacitor is linear its charge is

$$q(t) = Cv(t), \tag{5}$$

and so the current through the capacitor is

$$i(t) = \frac{\mathrm{d}}{\mathrm{d}t}(Cv(t)) = C\frac{\mathrm{d}v(t)}{\mathrm{d}t}. \tag{6}$$

Modeling a nonlinear capacitor by replacing C with $C(q)$ in (6) is in general a not good approach, because $C(q)$ varies with time. However, if C is not a strong function of q and q does not vary significantly with time, we can use it as approximation in our analysis.

In this paper, we assume that the voltage drops across capacitors presented in Fig 1 can be written as

$$v_{C_k}(t) = \frac{1}{C_k(q_k)} \int j_k(t)\mathrm{d}t = \frac{q_k(t)}{C_k(q_k)}, \tag{7}$$

where j_k denote the currents in the circuit, q_k represent the electric charges, C_k stand for the capacitances, $k = 1,2,\ldots,n$.

4. SYSTEM DESCRIPTION

Let us consider a nonlinear analog circuit shown in Fig. 1. The circuit consists of a set of resistors and capacitors that are connected together to form a network. The resistors and capacitors have in general nonlinear characteristics.

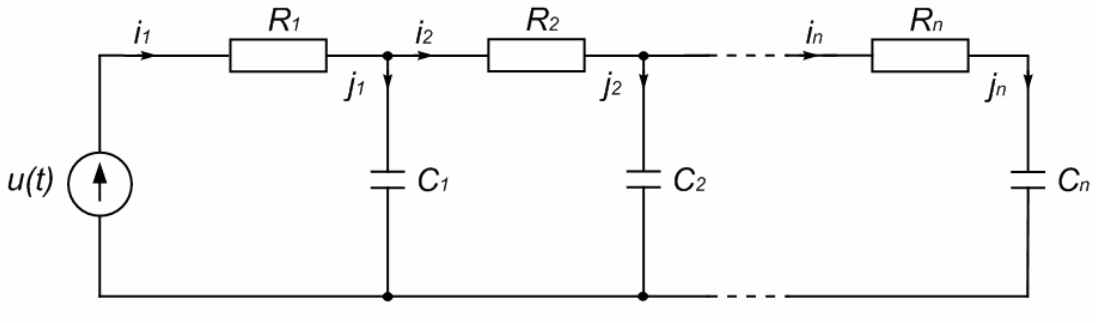

Fig. 1. RC ladder network
Rys. 1. Obwód drabinkowy typu RC

According to Kirchhoff's voltage law, the currents in the circuit are modeled by the equations:

$$R_1(i_1)i_1(t)+\frac{1}{C_1(q_1)}\int j_1(t)\mathrm{d}t = u(t), \tag{8}$$

$$R_2(i_2)i_2(t)+\frac{1}{C_2(q_2)}\int j_2(t)\mathrm{d}t = \frac{1}{C_1(q_1)}\int j_1(t)\mathrm{d}t\,, \tag{9}$$

and so on

$$R_n(i_n)i_n(t)+\frac{1}{C_n(q_n)}\int j_n(t)\mathrm{d}t = \frac{1}{C_{n-1}(q_{n-1})}\int j_{n-1}(t)\mathrm{d}t\,. \tag{10}$$

Applying Kirchhoff's current law to the circuit yields

$$i_1(t) = j_1(t)+i_2(t), \tag{11}$$

$$i_2(t) = j_2(t)+i_3(t), \tag{12}$$

and consequently

$$i_{n-1}(t) = j_{n-1}(t)+i_n(t), \tag{13}$$

$$i_n(t) = j_n(t). \tag{14}$$

Introducing the notation

$$\boldsymbol{i}(t)=[i_1(t) \quad i_2(t) \quad \ldots \quad i_n(t)]^{\mathrm{T}}, \tag{15}$$

$$\boldsymbol{j}(t)=[j_1(t) \quad j_2(t) \quad \ldots \quad j_n(t)]^{\mathrm{T}}, \tag{16}$$

$$\boldsymbol{p}(t)=[p_1(t) \quad p_2(t) \quad \ldots \quad p_n(t)]^{\mathrm{T}}, \tag{17}$$

$$\boldsymbol{q}(t)=[q_1(t) \quad q_2(t) \quad \ldots \quad q_n(t)]^{\mathrm{T}}, \tag{18}$$

$$\boldsymbol{i}(t)=\frac{\mathrm{d}\boldsymbol{p}(t)}{\mathrm{d}t}, \quad \boldsymbol{j}(t)=\frac{\mathrm{d}\boldsymbol{q}(t)}{\mathrm{d}t}, \tag{19}$$

and substituting the variables j_k in (8)-(10) by the expressions (11)-(14) we get

$$R_1(\dot{p}_1)\frac{\mathrm{d}p_1(t)}{\mathrm{d}t}+\frac{p_1(t)}{C_1(q_1)}-\frac{p_2(t)}{C_1(q_1)}=u(t), \tag{20}$$

$$R_2(\dot{p}_2)\frac{\mathrm{d}p_2(t)}{\mathrm{d}t}+\frac{p_2(t)}{C_2(q_2)}-\frac{p_3(t)}{C_2(q_2)}=\frac{p_1(t)}{C_1(q_1)}-\frac{p_2(t)}{C_1(q_1)}, \tag{21}$$

$$R_n(\dot{p}_n)\frac{\mathrm{d}p_n(t)}{\mathrm{d}t}+\frac{p_n(t)}{C_n(q_n)}=\frac{p_{n-1}(t)}{C_{n-1}(q_{n-1})}-\frac{p_n(t)}{C_{n-1}(q_{n-1})}. \tag{22}$$

Without loss of generality it can be assumed that

$$R_k(\dot{p}_k)=R_k(\dot{\boldsymbol{p}}), \tag{23}$$

for $k=1,2,\ldots,n$ and

$$C_k(q_k)=C_k(p_k,p_{k+1})=C_k(\boldsymbol{p}), \tag{24}$$

for $k=1,2,\ldots,n-1$, and

$$C_n(q_n)=C_n(p_n)=C_n(\boldsymbol{p}). \tag{25}$$

Then, the circuit's dynamic behavior can be modeled by the following equation:

$$\boldsymbol{R}(\dot{\boldsymbol{p}}(t))\dot{\boldsymbol{p}}(t)+\boldsymbol{C}(\boldsymbol{p}(t))\boldsymbol{p}(t)=\boldsymbol{B}u(t), \tag{26}$$

where

$$\boldsymbol{R}(\dot{\boldsymbol{p}})=\mathrm{diag}(R_1(\dot{\boldsymbol{p}}) \quad R_2(\dot{\boldsymbol{p}}) \quad \ldots \quad R_n(\dot{\boldsymbol{p}})), \tag{27}$$

$$\boldsymbol{C}(\boldsymbol{p})=\begin{bmatrix} d_1 & e_2 & 0 & \dots & 0 & 0 \\ f_2 & d_2 & e_3 & \dots & 0 & 0 \\ 0 & f_3 & d_3 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & d_{n-1} & e_n \\ 0 & 0 & 0 & \dots & f_n & d_n \end{bmatrix}, \tag{28}$$

where

$$d_1 = \frac{1}{C_1(\boldsymbol{p})}, \tag{29}$$

$$d_i = \frac{1}{C_{i-1}(\boldsymbol{p})} + \frac{1}{C_i(\boldsymbol{p})}, \tag{30}$$

$$e_i = f_i = -\frac{1}{C_{i-1}(\boldsymbol{p})}, \tag{31}$$

for $i = 2,3,\dots,n$,

$$\boldsymbol{B} = [1 \quad 0 \quad \dots \quad 0]^{\mathrm{T}}. \tag{32}$$

The initial condition $\boldsymbol{p}(0) = \boldsymbol{p}^{\boldsymbol{0}}$ is given as well.

The objective of the paper is to study the RC ladder network (26) under the following conditions:

(A1) The functions $R_i(\cdot)$, $C_i(\cdot)$, $i = 1,2,\dots,n$ are continuous with continuous derivatives in the set Ω, where $\Omega \subset \Re^n$ is a neighborhood of zero;

(A2) $R_i(\xi) > 0$, $C_i(\xi) > 0$ for $\xi \in \Omega$ and $i = 1,2,\dots,n$.

Lemma 1

$$\langle \xi, \boldsymbol{C}(\xi)\xi \rangle = \xi^{\mathrm{T}} \boldsymbol{C}(\xi)\xi > 0 \quad \text{for} \quad \xi \in \Omega \setminus \{\boldsymbol{0}\}. \tag{33}$$

Proof. The lemma is equivalent to the statement that for every $\xi \in \Omega$ the matrix $\boldsymbol{C}(\xi)$ is positive definite. It can be observed, that the matrix $\boldsymbol{C}(\xi)$ is a tridiagonal matrix and its determinant can be computed by the recursive formula [24, p. 54]. Using this formula, the leading principal minors M_k can be expressed as follows:

$$M_k = \frac{1}{C_1(\xi)} \frac{1}{C_2(\xi)} \cdots \frac{1}{C_k(\xi)}, \tag{34}$$

for $k = 2,3,\dots,n$. The leading principal minors M_k are positive, because $C_i(\xi) > 0$ for $i = 1,2,\dots,n$. This means that the matrix $\boldsymbol{C}(\xi)$ is positive definite. □

5. LINEAR DYNAMIC FEEDBACK CONTROL LAW

Let us consider the system (26) with linear dynamic feedback given in the following form:

$$\dot{w}(t)+aw(t)=bu(t),\quad w(0)=w^0, \tag{35}$$

$$u(t)=-k(w(t)+p_1(t))=-k\left(w(t)+\boldsymbol{B}^{\mathrm{T}}\boldsymbol{p}(t)\right), \tag{36}$$

where $w(t)\in\Re$, $a>0$, $b>0$, $k>0$.

The resulting closed-loop system becomes

$$\boldsymbol{R}(\dot{\boldsymbol{p}}(t))\dot{\boldsymbol{p}}(t)+\left(\boldsymbol{C}(\boldsymbol{p}(t))+k\boldsymbol{B}\boldsymbol{B}^{\mathrm{T}}\right)\boldsymbol{p}(t)+k\boldsymbol{B}w(t)=0, \tag{37}$$

$$\dot{w}(t)+(a+bk)w(t)+bk\boldsymbol{B}^{\mathrm{T}}\boldsymbol{p}(t)=0. \tag{38}$$

Theorem 1. *Suppose the assumptions (A1)-(A2) hold. Then the closed-loop system (37), (38) is locally asymptotically stable.*

Proof. The proof relies on imposing a suitable Lyapunov function for the closed-loop system (37), (38). Following candidate is considered:

$$V(\boldsymbol{p}(t),w(t))=\frac{a}{2b}w(t)^2+\int_{\boldsymbol{0}}^{\boldsymbol{p}(t)}(\boldsymbol{C}(\boldsymbol{\xi})\boldsymbol{\xi})^{\mathrm{T}}\,\mathrm{d}\boldsymbol{\xi}+\frac{1}{2}k\left(w(t)+\boldsymbol{B}^{\mathrm{T}}\boldsymbol{p}(t)\right)^2. \tag{39}$$

The integral in the formula (39) denotes a line integral along the straight line in the space $\Re^n$ from the beginning point $\boldsymbol{0}$ to the ending point $\boldsymbol{p}(t)$. It can be proven by using Lemma 1 that this integral is positive for $\boldsymbol{\xi}\neq\boldsymbol{0}$ and equals zero for $\boldsymbol{\xi}=\boldsymbol{0}$. The derivative of the functional (39) becomes

$$\begin{aligned}\dot{V}(\boldsymbol{p}(t),w(t))&=\frac{a}{b}w(t)\dot{w}(t)+(\boldsymbol{C}(\boldsymbol{p}(t))\boldsymbol{p}(t))^{\mathrm{T}}\dot{\boldsymbol{p}}(t)+k\left(w(t)+\boldsymbol{B}^{\mathrm{T}}\boldsymbol{p}(t)\right)\left(\dot{w}(t)+\boldsymbol{B}^{\mathrm{T}}\dot{\boldsymbol{p}}(t)\right)\\&=\frac{a}{b}w(t)\dot{w}(t)+\boldsymbol{p}(t)^{\mathrm{T}}\boldsymbol{C}(\boldsymbol{p}(t))\dot{\boldsymbol{p}}(t)-u(t)\left(\dot{w}(t)+\boldsymbol{B}^{\mathrm{T}}\dot{\boldsymbol{p}}(t)\right).\end{aligned} \tag{40}$$

Evaluating the time derivative of V along the solution of the system (37), (38) gives

$$\begin{aligned}\dot{V}(\boldsymbol{p}(t),w(t))=\frac{a}{b}w(t)(bu(t)-aw(t))+\boldsymbol{p}(t)^{\mathrm{T}}\boldsymbol{C}(\boldsymbol{p}(t))\boldsymbol{R}(\dot{\boldsymbol{p}}(t))^{-1}(\boldsymbol{B}u(t)-\boldsymbol{C}(\boldsymbol{p}(t))\boldsymbol{p}(t))\\-u(t)(bu(t)-aw(t))-u(t)\boldsymbol{B}^{\mathrm{T}}\boldsymbol{R}(\dot{\boldsymbol{p}}(t))^{-1}(\boldsymbol{B}u(t)-\boldsymbol{C}(\boldsymbol{p}(t))\boldsymbol{p}(t)).\end{aligned} \tag{41}$$

After some elementary calculations

$$\begin{aligned}\dot{V}(\boldsymbol{p}(t),w(t))=-\frac{1}{b}(bu(t)-aw(t))^2\\-(\boldsymbol{B}u(t)-\boldsymbol{C}(\boldsymbol{p}(t))\boldsymbol{p}(t))^{\mathrm{T}}\boldsymbol{R}(\dot{\boldsymbol{p}}(t))^{-1}(\boldsymbol{B}u(t)-\boldsymbol{C}(\boldsymbol{p}(t))\boldsymbol{p}(t)),\end{aligned} \tag{42}$$

it can be seen that

$$\dot{V}(\boldsymbol{p}(t),w(t)) = -\frac{1}{b}\dot{w}(t)^2 - \dot{\boldsymbol{p}}(t)^{\mathrm{T}}\boldsymbol{R}(\dot{\boldsymbol{p}}(t))\dot{\boldsymbol{p}}(t). \tag{43}$$

It should be noted that $V(\boldsymbol{p},w) > 0$ for $\mathrm{col}(\boldsymbol{p},w) \in \Omega_c \setminus \{\boldsymbol{0}\}$, $V(\boldsymbol{0},0) = 0$ and $\dot{V}(\boldsymbol{p},w) \le 0$ for $\mathrm{col}(\boldsymbol{p},w) \in \Omega_c$, where Ω_c is a compact set defined as follows:

$$\Omega_c = \{\boldsymbol{z} = \mathrm{col}(\boldsymbol{p},w):\ \boldsymbol{p} \in \Omega, w \in \Re, V(\boldsymbol{z}) < c\}, \tag{44}$$

c is a real positive number, $\mathrm{col}(\boldsymbol{p},w) = \left[\boldsymbol{p}^{\mathrm{T}} \quad w^{\mathrm{T}}\right]^{\mathrm{T}}$.

The next part of the proof is based on the LaSalle's invariance principle [4]. According to this principle, the trajectories of the closed-loop system (37), (38) starting in Ω_c enter the largest invariant set in S, where

$$S = \{\boldsymbol{z} \in \Omega_c :\ \dot{V}(\boldsymbol{z}) = 0\}. \tag{45}$$

From $\dot{V}(\boldsymbol{z}) = 0$ it follows that $\dot{w}(t) = 0$ and $\dot{\boldsymbol{p}}(t) = 0$. This implies that $w(t) = w^0$ and $\boldsymbol{p}(t) = \boldsymbol{p}^{\boldsymbol{0}}$. Using this result in (26) and (35) yields

$$\boldsymbol{C}(\boldsymbol{p}^{\boldsymbol{0}})\boldsymbol{p}^{\boldsymbol{0}} = \frac{a}{b}\boldsymbol{B}w^0. \tag{46}$$

It is easy to show that the equation (46) has solution only for $w^0 = 0$ because $a > 0$, $b > 0$ and Lemma 1 is valid. Thus $w(t) = 0$ and $\boldsymbol{p}(t) = 0$ for all $t > 0$. This means that $S = \{\boldsymbol{0}\}$ contains only the zero solution, and by LaSalle's principle, the origin $0 \in \Re^{n+1}$ is asymptotically stable (in the Lyapunov sense). □

6. NONLINEAR DYNAMIC FEEDBACK CONTROL LAW

Let us consider the system (26) with nonlinear dynamic feedback given in the following form:

$$\dot{w}(t) + aw(t) = bu(t), \quad w(0) = w^0, \tag{47}$$

$$u(t) = -\frac{1}{K(t)}(w(t) + p_1(t)) = -\frac{1}{K(t)}\left(w(t) + \boldsymbol{B}^{\mathrm{T}}\boldsymbol{p}(t)\right), \tag{48}$$

where

$$K(t) = K_0 + \gamma\left(w(t) + \boldsymbol{B}^{\mathrm{T}}\boldsymbol{p}(t)\right)^2, \tag{49}$$

and $K_0 > 0$, $\gamma > 0$.

The closed-loop system is given by the following equations:

$$\boldsymbol{R}(\dot{\boldsymbol{p}}(t))\dot{\boldsymbol{p}}(t)+\left(\boldsymbol{C}(\boldsymbol{p}(t))+\frac{1}{K(t)}\boldsymbol{B}\boldsymbol{B}^{\mathrm{T}}\right)\boldsymbol{p}(t)+\frac{1}{K(t)}\boldsymbol{B}w(t)=0\,, \tag{50}$$

$$\dot{w}(t)+\left(a+\frac{b}{K(t)}\right)w(t)+\frac{b}{K(t)}\boldsymbol{B}^{\mathrm{T}}\boldsymbol{p}(t)=0\,. \tag{51}$$

Theorem 2. *Suppose the assumptions (A1)-(A2) hold. Then the closed-loop system (50), (51) is locally asymptotically stable.*

Proof. The proof can be divided into two parts. First, it can be proved that

$$V(\boldsymbol{p}(t),w(t))=\frac{a}{2b}w(t)^2+\int_{\boldsymbol{0}}^{\boldsymbol{p}(t)}(\boldsymbol{C}(\boldsymbol{\xi})\boldsymbol{\xi})^{\mathrm{T}}\,\mathrm{d}\boldsymbol{\xi}+\frac{1}{2\lambda}\ln\frac{K(t)}{K_0} \tag{52}$$

is the Lyapunov function for the system (50), (51). Then, following the method as in the previous section, it can be concluded by LaSalle's theorem [4] that the trajectories tend to the origin $\{\boldsymbol{0}\}$ as t goes to infinity. □

7. SIMULATION EXAMPLE

RC circuits are useful, simple and robust passive electric circuits. They play integral roles in everyday electronic equipment such as traffic lights, pacemakers, audio and radio equipment. While their applications are numerous and varied, they are mostly used for their signal filtering capabilities and precise timing abilities (for example: Wien-bridge oscillator, phase-shift oscillator, high pass and low pass filters, etc.). In this section, a simple RC circuit with nonlinear elements is analyzed to verify mathematical formulation from the previous sections.

Let us consider the circuit presented in Fig. 2.

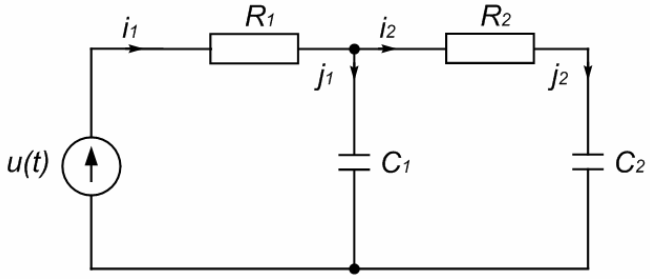

Fig. 2. RC ladder network with $n = 2$
Rys. 2. Obwód drabinkowy typu RC dla $n = 2$

The resistances R_1 and R_2 are linear

$$R_1 = 0.2\,[\Omega], \quad R_2 = 0.3\,[\Omega]. \tag{53}$$

The characteristics of the capacitances C_1 and C_2 are nonlinear and given in the analytic form:

$$C_1(\boldsymbol{p}) = 50\exp\left(-0.3(p_1 - p_2)^2\right), \tag{54}$$

$$C_2(\boldsymbol{p}) = 100\exp\left(-0.5p_2^2\right). \tag{55}$$

The dynamics of electric charge flow in the circuit can be described by the equation

$$\boldsymbol{R}(\dot{\boldsymbol{p}}(t))\dot{\boldsymbol{p}}(t) + \boldsymbol{C}(\boldsymbol{p}(t))\boldsymbol{p}(t) = \boldsymbol{B}u(t), \tag{56}$$

where

$$\boldsymbol{R}(\dot{\boldsymbol{p}}) = \begin{bmatrix} R_1 & 0 \\ 0 & R_2 \end{bmatrix}, \tag{57}$$

$$\boldsymbol{C}(\boldsymbol{p}) = \begin{bmatrix} C_1(\boldsymbol{p})^{-1} & -C_1(\boldsymbol{p})^{-1} \\ -C_1(\boldsymbol{p})^{-1} & C_1(\boldsymbol{p})^{-1} + C_2(\boldsymbol{p})^{-1} \end{bmatrix}, \tag{58}$$

$$\boldsymbol{B} = [1 \quad 0]^{\mathrm{T}}, \tag{59}$$

$$\boldsymbol{p}(t) = [p_1(t) \quad p_2(t)]^{\mathrm{T}}. \tag{60}$$

Here $\boldsymbol{p}(t)$ represents the vector of electric charges, $u(t)$ is the control voltage, $\boldsymbol{p}(t) \in \Re^2$, $u(t) \in \Re$, $t > 0$. The voltage of the power source is measured in volts [V], the resistance of the resistors is measured in ohms [Ω], the capacitance of the capacitors is measured in farads [F] and the charge across the capacitors is measured in coulombs [C]. The following initial conditions are used for the differential equation (56):

$$p_1(0) = 0.3, \quad p_2(0) = 0.2. \tag{61}$$

Let us introduce one-dimensional parallel compensator

$$\dot{w}(t) + 0.2w(t) = 0.5u(t), \quad w(0) = 0, \tag{62}$$

and design the controller

$$u(t) = -0.5(w(t) + p_1(t)). \tag{63}$$

It is easy to check that the assumptions (A1)-(A2) hold as well as Lemma 1 is valid. According to Theorem 1 the closed-loop system (56), (62), (63) is asymptotically stable. The trajectories of the open-loop system (dot line) and closed-loop system (solid line) are shown in Figs. 3-5.

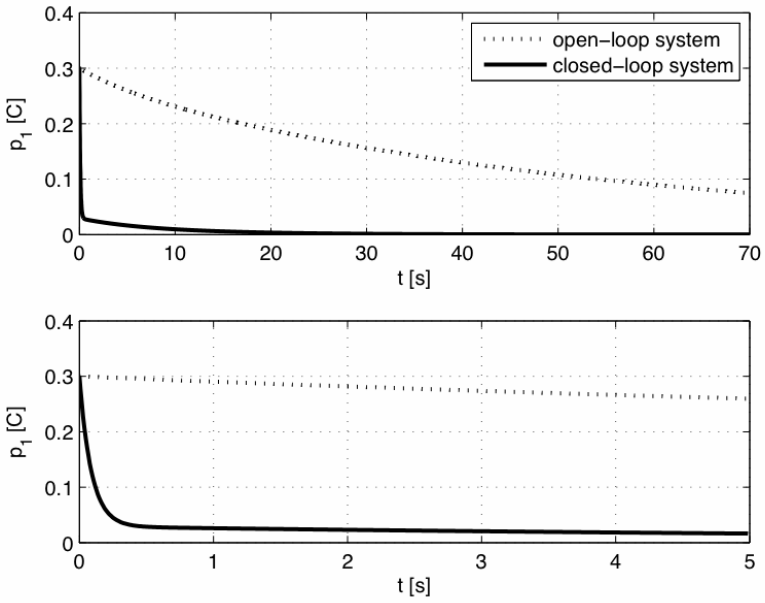

Fig. 3. The electric charge $p_1(t)$ in the open-loop circuit (dot line) and closed-loop circuit (solid line). The same simulation results are shown in different time windows

Rys. 3. Dynamika zmian ładunku elektrycznego $p_1(t)$ w układzie otwartym (linia przerywana) oraz w układzie zamkniętym (linia ciągła). Ten sam przebieg jest pokazany w różnych oknach czasowych

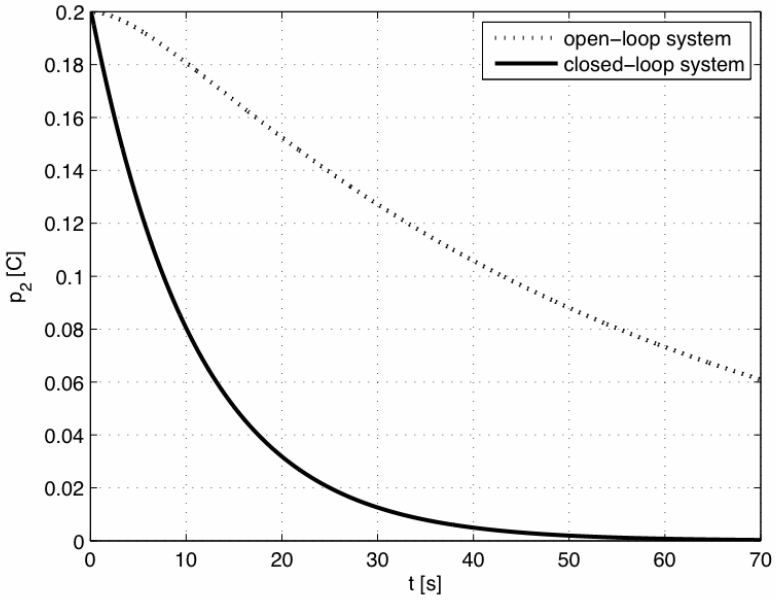

Fig. 4. The electric charge $p_2(t)$ in the open-loop circuit (dot line) and closed-loop circuit (solid line)

Rys. 4. Dynamika zmian ładunku elektrycznego $p_2(t)$ w układzie otwartym (linia przerywana) oraz w układzie zamkniętym (linia ciągła)

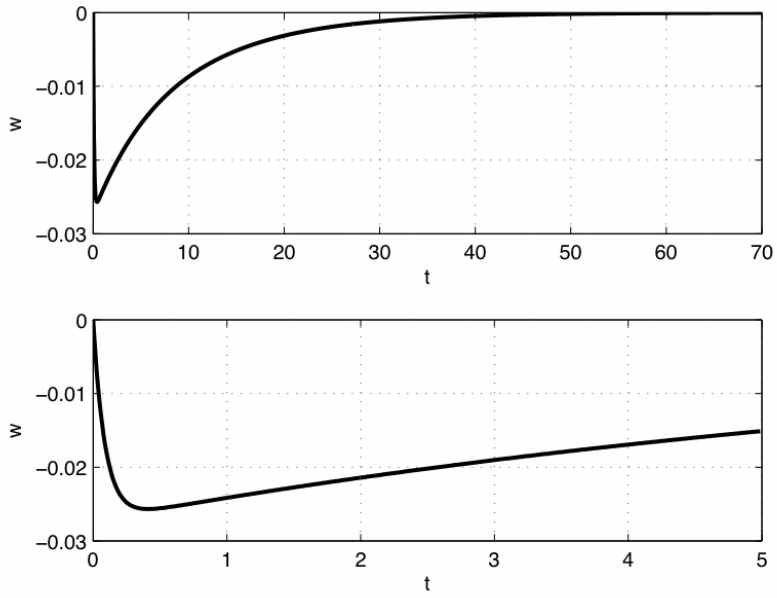

Fig. 5. The state variable $w(t)$ of the compensator. The same simulation results are shown in different time windows

Rys. 5. Trajektoria $w(t)$ kompensatora dynamicznego. Ten sam przebieg jest pokazany w różnych oknach czasowych

8. CONCLUSIONS

In the paper, linear and nonlinear techniques for stabilization of a class of nonlinear RC ladder networks have been investigated. It has been shown that the system is asymptotically stable when linear dynamic feedback is applied. The asymptotic stability of the closed-loop system has been proved by LaSalle's invariance principle using appropriate Lyapunov function. The similar results have been obtained with nonlinear dynamic feedback. Numerical calculations and computer simulations have been performed in the MathWorks™ MATLAB®/Simulink® environment to show the effectiveness of the proposed methods.

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Omówienie

W pracy rozważono zagadnienie stabilizacji dla wybranej klasy układów drabinkowych typu RC. Wybrana klasa układów obejmuje obwody elektryczne, składające się z rezystorów i kondensatorów o nieliniowych charakterystykach. Wartość rezystancji dla rezystorów nieliniowych jest funkcją prądu. Nieliniowy kondensator charakteryzuje się tym, że jego pojemność zależy od napięcia występującego na okładzinach kondensatora lub od ładunku zgromadzonego na tych okładzinach. W rzeczywistości wszystkie obwody elektryczne są nieliniowe, gdyż wszystkie elementy rzeczywiste wykazują cechy nieliniowości.

Dynamika układu drabinkowego RC może być modelowana matematycznie za pomocą nieliniowych równań różniczkowych zwyczajnych. Na podstawie tego modelu skonstruowano dynamiczne sprzężenia zwrotne, które asymptotycznie stabilizują system. Zaproponowano zarówno liniowe, jak i nieliniowe regulatory. Własność asymptotycznej stabilności układu zamkniętego została pokazana z wykorzystaniem odpowiednich funkcjonałów Lapunowa oraz twierdzenia LaSalle’a. Wyniki teoretyczne zostały zweryfikowane przez obliczenia numeryczne i symulacje komputerowe.

Obwody nieliniowe są powszechnie stosowane w urządzeniach elektrycznych i elektronicznych. Dzięki elementom nieliniowym jest możliwe realizowanie takich czynności, jak prostowanie, stabilizacja napięcia i prądu, modulacja i detekcja sygnałów, wytwarzanie sygnałów o różnych kształtach itp. Analiza układów nieliniowych jest trudna i bardzo często przybliżona. Metoda badania takich układów polega zazwyczaj na linearyzacji poszczególnych elementów lub analizie numerycznej. Dlatego też na szczególną uwagę zasługują rozwiązania analityczne, które uwzględniają nieliniowości występujące w układzie.