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MAGNETIC FIELD MITIGATION FROM SAGGING POWER LINES USING A SAGGING PASSIVE LOOP

Summary. The study presents in a tutorial manner methods of the calculation of magnetic fields in vicinity of catenary electric power lines without and with mitigation loops. A solution for modelling magnetic fields produced by sagging conductor, described by the catenary equation, is proposed. It is assumed that the equation of the catenary exactly describes the line sag and the influence of currents induced in the earth on the distribution of power line magnetic field is neglected. Moreover the mitigation effects due to the passive loop are also investigated, whereas the mitigation loop can be treated as a rectangular loop (two-conductor closed mitigation loop) horizontal located under the power line or as a closed loop with two sagging conductors.

Keywords: magnetic field, overhead power line, earth return, catenary, passive loop

OSŁABIENIE POLA MAGNETYCZNEGO LINII ELEKTROENERGETYCZNEJ Z ZASTOSOWANIEM PASYWNYCH EKRANÓW Z UWZGLĘDNIENIEM ZWISU PRZEWODÓW

Streszczenie. W pracy zaprezentowano metody wyznaczania pola magnetycznego pod linią elektroenergetyczną w przypadku zastosowania pasywnego ekranu – pętli utworzonej przez parę przewodów podwieszonych pod linią, z uwzględnieniem zwisu przewodów linii i pętli. Zwis przewodów opisany jest równaniem łańcuchowym. Pomija się wpływ prądów indukowanych w ziemi na rozkład pola magnetycznego. Wyprowadzono wzory analityczne i omówiono sposób wyznaczenia efektu ekranowania, w pierwszej kolejności dla przewodów pętli pasywnej bez uwzględnienia zwisu, a następnie z jego uwzględnieniem.

Słowa kluczowe: pole magnetyczne, napowietrzna linia elektroenergetyczna, obwód ziemnopowrotny, linia łańcuchowa, pętla pasywna

1. INTRODUCTION

Transmission of the electric power is accompanied with generation of low – frequency electromagnetic fields. Nowadays of special concern is the possibility of detrimental environmental effects arising from the electrical and magnetic fields formed adjacent to the

overhead transmission lines. These fields may affect both operation of near electric and electronic devices and appliances and also various living organisms.

The topic of mitigating the magnetic fields produced by overhead power lines is gaining more significance in recent years. In this context, efforts are continuously being done in order to maximize the utilization of the available line corridors without exceeding the tolerable limits of the lines' magnetic fields.

There are several possibilities to mitigate the field from existing or new overhead lines, e.g.: increasing the height of conductors, phase rearrangement, compaction, splitting of phases, underground cables, gas-insulated lines, passive shields (ferromagnetic and conductive), etc.

During the last decades, the use of conductive shields (passive non-compensated or series capacitor compensated loops) to mitigate extremely low frequency magnetic fields generated from power lines has been proposed [1 - 15]. Their behavior principle is based on the electromagnetic induction law: time varying, primary magnetic fields, generated by AC sources, induce electromotive forces driving loops currents – additional field sources, which modify and reduce the primary magnetic field.

The power line conductors are periodic catenaries, the sag of which depends on individual characteristics of the line and on environmental conditions. The effect of the catenary on the amplitude of the magnetic field can be significant in some cases. In [16] a solution is proposed for modeling magnetic fields produced by sagging conductors. It is assumed that the equation of the catenary exactly describes the line sag and the influence of currents induced in the earth on the distribution of power line magnetic field is neglected.

In the paper the mitigation effects due to the passive loop are also investigated, whereas the mitigation loop can be treated as a rectangular loop (two-conductor closed mitigation loop) horizontal located under the power line or as a closed loop with two sagging conductors.

2. COMPUTATION OF THE MAGNETIC FIELD OF CATENARY HIGH VOLTAGE POWER LINES

2. 1. Unmitigated magnetic field under sagging conductor

Consider the electromagnetic field in the air produced by a current carrying conductor hanging over the earth surface (x, y plane), as shown in Fig. 1. It is assumed that the current flows along the $0x$ axis, that the length of the span is L, that the maximum and minimum heights of catenary are H and h respectively and the displacement currents both in the air and in the earth are neglected. Furthermore, it is supposed that the influence of the induced earth return currents on the magnetic field has negligible practical importance, as shown in [16].

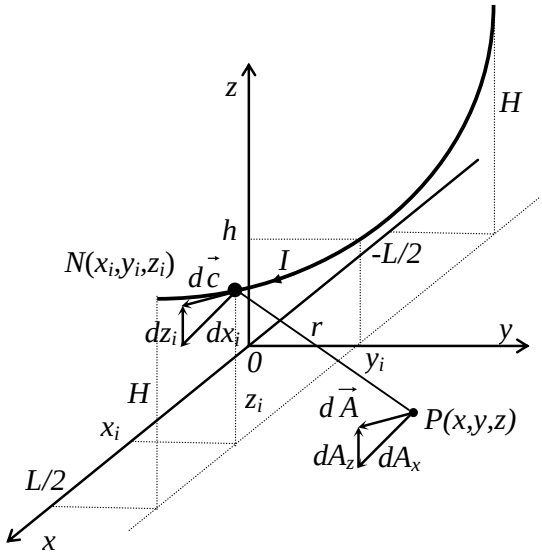

Fig. 1. Current carrying sagging conductor above the earth surface
Rys. 1. Napowietrzny przewód z prądem

The analytical calculation of the incident magnetic field in the free space (air) generated by a time-harmonic current I can be obtained by the application of the Poisson equation:

$$\Delta\vec{A} = -\mu_0 \vec{J} \tag{1}$$

where J denotes the current density.

It can be shown, that, assuming that the conductor is very thin, the solution of the eqn. (1) takes the form:

$$\vec{A} = \frac{I\mu_0}{4\pi}\int_c \frac{\overrightarrow{dc}}{r} \tag{2}$$

where the vector element $d\vec{c}$ coincides with the direction of the current I, r is the distance between the source point $N(x_i, y_i, z_i)$ and the observation point $P(x, y, z)$ and the distance r is given by:

$$r = \sqrt{(x - x_i)^2 + (y - y_i)^2 + (z - z_i)^2} \tag{3}$$

Since the modelled curve is located in the x, z plane the elementary vector $d\vec{c}$ in the Cartesian co-ordinates system can be written as:

$$d\vec{c} = \vec{e}_x dx_i + \vec{e}_z dz_i \tag{4}$$

A sagging power line conductor has a form of a catenary curve. The approximating equation of the catenary can be written in the form [16]:

$$z_i = h + \alpha\left[\cosh(\frac{x_i}{\alpha}) - 1\right] \tag{5}$$

where α - parameter iterative obtained from the eqn.(6):

$$H = h + \alpha\left[\cosh(\frac{L}{2\alpha}) - 1\right] \tag{6}$$

Taking into account the relationship (5) the elementary vector $d\vec{c}$ can be written in the form:

$$d\vec{c} = \vec{e}_x dx_i + \vec{e}_z \sinh(\frac{x_i}{\alpha}) dx_i \tag{7}$$

The *x*- and *z*-components of the vector potential become according to the formula (2):

$$A_x = \frac{I\mu_0}{4\pi}\int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{dx_i}{\sqrt{(x-x_i)^2 + (y-y_i)^2 + \left(z - h - \alpha\left[\cosh(\frac{x_i}{\alpha}) - 1\right]\right)^2}} \tag{8}$$

$$A_z = \frac{I\mu_0}{4\pi}\int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{\sinh(\frac{x_i}{\alpha})\, dx_i}{\sqrt{(x-x_i)^2 + (y-y_i)^2 + \left(z - h - \alpha\left[\cosh(\frac{x_i}{\alpha}) - 1\right]\right)^2}} \tag{9}$$

The magnetic field density can be obtained from the relation (10):

$$\vec{B} = rot\vec{A} \tag{10}$$

Taking into account the eqn.(8) and (9) the *x*, *y* ,*z* -components of the magnetic field density become:

$$B_x = \frac{I\mu_0}{4\pi}\int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{\sinh(\frac{x_i}{\alpha})(y-y_i)\, dx_i}{\left[(x-x_i)^2 + (y-y_i)^2 + \left(z - h - \alpha\left[\cosh(\frac{x_i}{\alpha}) - 1\right]\right)^2\right]^{\frac{3}{2}}} \tag{11}$$

$$B_y = -\frac{I\mu_0}{4\pi}\int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{\left[\sinh(\frac{x_i}{\alpha})(x-x_i) - (z-z_i)\right] dx_i}{\left[(x-x_i)^2 + (y-y_i)^2 + \left(z - h - \alpha\left[\cosh(\frac{x_i}{\alpha}) - 1\right]\right)^2\right]^{\frac{3}{2}}} \tag{12}$$

$$B_z = -\frac{I\mu_0}{4\pi}\int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{(y-y_i)\, dx_i}{\left[(x-x_i)^2 + (y-y_i)^2 + \left(z - h - \alpha\left[\cosh(\frac{x_i}{\alpha}) - 1\right]\right)^2\right]^{\frac{3}{2}}} \tag{13}$$

Based on the above formulas for a single span single conductor catenary, in presence of many current sources (current carrying conductors), the superposition can be applied.

The magnitude of the magnetic flux density can be calculated from the relationship:

$$B = \sqrt{B_x^2 + B_y^2 + B_z^2} \tag{14}$$

The most interesting location of the observation point regarding the value of the magnetic flux density produced by a current in a sagging conductor is the point located under the conductor at the midspan. A magnetic flux density distribution on the earth surface is depicted in Fig. 2. The calculation has been carried out for parameters: L = 400 m, H = 10 m, h = 6 m; the unit current I = 1 A is chosen as an example and based on the fact that there exist a direct proportion between the current and the magnetic field.

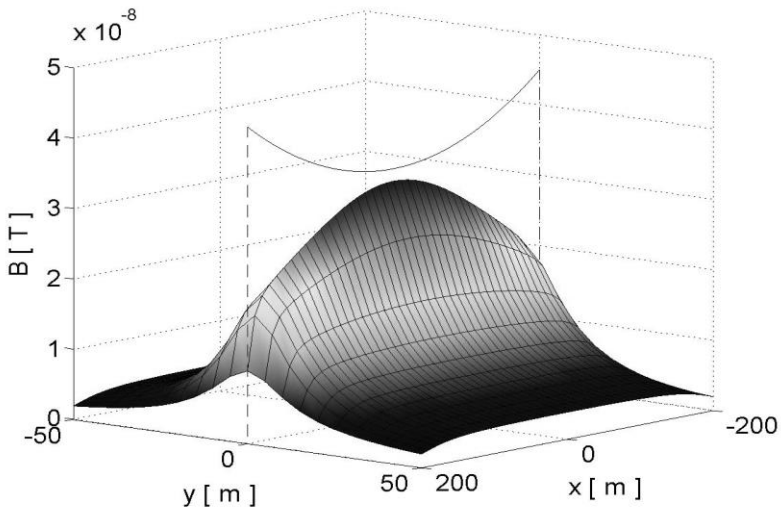

Fig. 2. Magnetic flux density distribution [16]
Rys. 2. Rozkład indukcji magnetycznej [16]

The total x, y and z components of the unmitigated magnetic flux density in the vicinity of the overhead 3-phase power line with phase conductors (n) and earth wires (m) can be obtained by superposition, according to the equations (15), (16) and (17):

$$B_x = \sum_{i=1}^{n+m} B_{xi} \tag{15}$$

$$B_y = \sum_{i=1}^{n+m} B_{yi} \tag{16}$$

$$B_z = \sum_{i=1}^{n+m} B_{zi} \tag{17}$$

2.2. Current in a mitigation loop

Consider next the magnetic flux passing through a surface of the horizontal, rectangular loop located underneath a sagging overhead conductor with a current, as in Fig. 3.

Magnetic flux penetrating through the loop (Fig.3) is obtained by applying the equation:

$$\Phi = \oint_l \vec{A} \cdot \vec{dl} \tag{18}$$

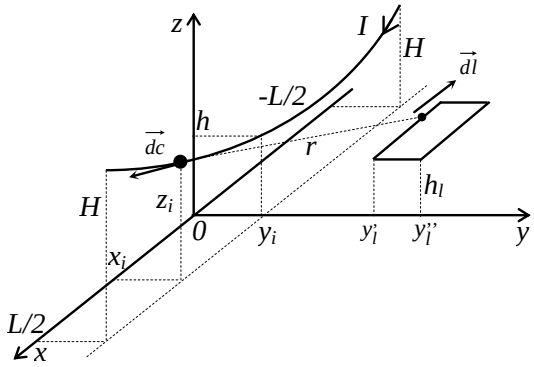

Fig. 3. Current carrying sagging conductor and a loop
Rys. 3. Napowietrzny przewód z prądem oraz pętla

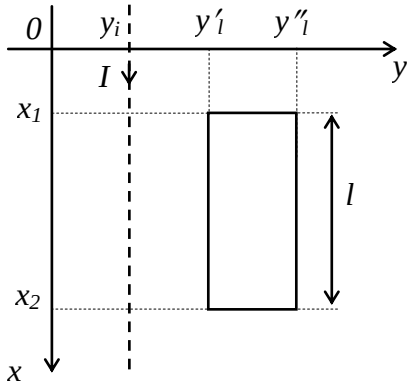

Fig. 4. Current carrying sagging conductor and a loop – top view
Rys. 4. Napowietrzny przewód z prądem oraz pętla – widok z góry

In eqn.(18) only the x – component of the vector potential $\vec{A}$ contributes to the value of the integral since:

$$\vec{A}\cdot\vec{dl} = A_x dl_x + A_y dl_y + A_z dl_z \tag{19}$$

and $A_y = 0$ and $dl_z = 0$.

When the relationship (8) is taken into account, the resultant magnetic flux will be:

$$\Phi = \frac{I\mu_0}{4\pi}\left|\left\{\begin{array}{l}\displaystyle\int_{x_1}^{x_2}\int_{-\frac{L}{2}}^{\frac{L}{2}}\frac{dx_i dx}{\sqrt{(x-x_i)^2+(y_l''-y_i)^2+\left(h_l-h-\alpha\left[\cosh(\frac{x_i}{\alpha})-1\right]\right)^2}}+\\ \displaystyle-\int_{x_1}^{x_2}\int_{-\frac{L}{2}}^{\frac{L}{2}}\frac{dx_i dx}{\sqrt{(x-x_i)^2+(y_l'-y_i)^2+\left(h_l-h-\alpha\left[\cosh(\frac{x_i}{\alpha})-1\right]\right)^2}}\end{array}\right\}\right| \tag{20}$$

It easy to show using the Neumann's formula

$$M = \frac{\mu_0}{4\pi} \oint_c \oint_l \frac{\overrightarrow{dc} \cdot \overrightarrow{dl}}{r} \tag{21}$$

that

$$M = \frac{\mu_0}{4\pi} \left\| \left\{ \begin{array}{l} \int_{x_1}^{x_2} \int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{dx_i dx}{\sqrt{(x-x_i)^2 + (y_l'' - y_i)^2 + \left(h_l - h - \alpha\left[\cosh(\frac{x_i}{\alpha}) - 1\right]\right)^2}} + \\ - \int_{x_1}^{x_2} \int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{dx_i dx}{\sqrt{(x-x_i)^2 + (y_l' - y_i)^2 + \left(h_l - h - \alpha\left[\cosh(\frac{x_i}{\alpha}) - 1\right]\right)^2}} \end{array} \right\} \right\| \tag{22}$$

expresses the mutual inductance between the sagging conductor and the loop.

The electromotive force induced in the loop can be obtained from the relation (23):

$$\mathrm{E} = -j\omega\Phi \tag{23}$$

Thus:

$$\mathrm{E} = -\frac{j\omega I\mu_0}{4\pi} \left\| \left\{ \begin{array}{l} \int_{x_1}^{x_2} \int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{dx_i dx}{\sqrt{(x-x_i)^2 + (y_l'' - y_i)^2 + \left(h_l - h - \alpha\left[\cosh(\frac{x_i}{\alpha}) - 1\right]\right)^2}} + \\ - \int_{x_1}^{x_2} \int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{dx_i dx}{\sqrt{(x-x_i)^2 + (y_l' - y_i)^2 + \left(h_l - h - \alpha\left[\cosh(\frac{x_i}{\alpha}) - 1\right]\right)^2}} \end{array} \right\} \right\| \tag{24}$$

The loop current can be obtained from the relation:

$$I_{loop} = -\frac{j\omega I\mu_0}{4\pi Z_s} \left\| \left\{ \begin{array}{l} \int_{x_1}^{x_2} \int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{dx_i dx}{\sqrt{(x-x_i)^2 + (y_l'' - y_i)^2 + \left(h_l - h - \alpha\left[\cosh(\frac{x_i}{\alpha}) - 1\right]\right)^2}} + \\ - \int_{x_1}^{x_2} \int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{dx_i dx}{\sqrt{(x-x_i)^2 + (y_l' - y_i)^2 + \left(h_l - h - \alpha\left[\cosh(\frac{x_i}{\alpha}) - 1\right]\right)^2}} \end{array} \right\} \right\| \tag{25}$$

whith the loop self-impedance Z_s calculated according the eqn.(26) – (28) [19, 20]:

$$Z_s = j\omega \frac{\mu_0}{2\pi} \ln \frac{2(h_k + p)}{r_k} \tag{26}$$

where r_k is the radius of the conductor k and

$$p = \frac{1}{\sqrt{j\omega\mu_0\gamma}} = \frac{\delta_s}{2}(1-j) \tag{27}$$

and the penetration depth

$$\delta_s = \frac{1}{\sqrt{\pi f \mu_0 \gamma}} \tag{28}$$

2.3. Mitigated magnetic field under sagging power lines using a passive loop

Based on the above formulas for a single span single conductor catenary, in presence of many current sources (current carrying conductors), the superposition can be applied.

The magnitude of the magnetic flux density can be next calculated from the relationship:

$$B = \sqrt{B_x^2 + B_y^2 + B_z^2} \tag{29}$$

The total x, y and z components of the magnetic flux density in the vicinity of the overhead sagging 3-phase power line with phase conductors (n) and earth wires (m) and auxiliary conductors (k) forming the loop can be obtained by superposition, according to the equations (30) - (32):

$$B_x = \sum_{i=1}^{n+m+k} B_{xi} \tag{30}$$

$$B_y = \sum_{i=1}^{n+m+k} B_{yi} \tag{31}$$

$$B_z = \sum_{i=1}^{n+m+k} B_{zi} \tag{32}$$

It should be noted, that the magnetic flux density produced by the *k–th* auxiliary current has to be approximately obtained according to [16]:

$$B_{yk\,u}(y,z) = -\frac{\mu_0 I_{u\,loop}}{2\pi} \frac{z-h_k}{(z-h_k)^2 + (y-y_k)^2} \tag{33}$$

$$B_{zk\,u}(y,z) = \frac{\mu_0 I_{u\,loop}}{2\pi} \frac{y-y_k}{(z-h_k)^2 + (y-y_k)^2} \tag{34}$$

It follows from [16], that in the calculations of the magnetic field in the observation point located at the midspan, the influence of the currents flowing in the nearby spans can be neglected. On the other hand, in the magnetic field calculations near power line towers only the influence of the current in the nearest span has to be taken into account.

2.4. Current in a sagging loop under sagging conductor

Consider next the mutual inductance between a sagging overhead conductor with a current I_k and a sagging loop, as in Fig. 5. All sagging conductors are located in the *x*, *z* plane.

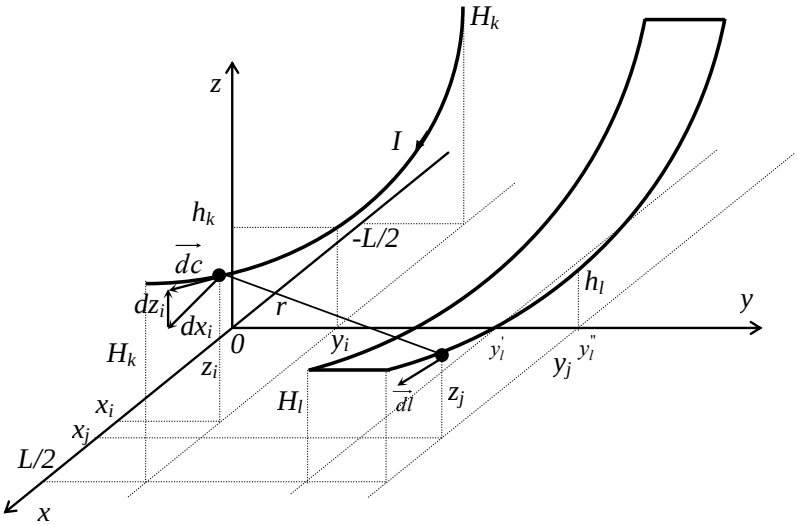

Fig. 5. Sagging loop under sagging conductor
Rys. 5. Przewód z prądem i pętla

The approximating equation of the sagging conductor can be written now in the form:

$$z_i = h_k + \alpha_k \left[\cosh(\frac{x_i}{\alpha_k}) - 1 \right] \tag{35}$$

where α_k - parameter iterative obtained from the eqn.(36):

$$H_k = h_k + \alpha_k \left[\cosh(\frac{L}{2\alpha_k}) - 1 \right] \tag{36}$$

The elementary vector $d\vec{c}$ in the Cartesian co-ordinates system can be written as:

$$d\vec{c} = \vec{e}_x dx_i + \vec{e}_z dz_i = \vec{e}_x dx_i + \vec{e}_z \sinh(\frac{x_i}{\alpha_k}) dx_i \tag{37}$$

Assuming the span length of the loop is also *L* but other parameters of the catenary curve describing the loop-conductor differ from parameters of the overhead conductors (Fig. 5), the approximating equation of the sagging loop-conductor can be written in the form:

$$z_j = h_l + \alpha_l \left[\cosh(\frac{x_j}{\alpha_l}) - 1 \right] \tag{38}$$

where α_l - parameter iterative obtained from the eqn.(39):

$$H_l = h_l + \alpha_l \left[\cosh(\frac{L}{2\alpha_l}) - 1 \right] \tag{39}$$

The elementary vector $d\vec{l}$ can be written as:

$$d\vec{l} = \vec{e}_x dx_j + \vec{e}_z dz_j = \vec{e}_x dx_j + \vec{e}_z \sinh(\frac{x_j}{\alpha_l}) dx_j \tag{40}$$

The scalar product of elementary vectors has therefore the form:

$$\overrightarrow{dc} \cdot \overrightarrow{dl} = dx_i dx_j + \sinh(\frac{x_i}{\alpha_k}) dx_i \sinh(\frac{x_j}{\alpha_l}) dx_j = [1 + \sinh(\frac{x_i}{\alpha_k}) \sinh(\frac{x_j}{\alpha_l})] dx_i dx_j \tag{41}$$

Taking into account that the distance r between the source point and the observation point is:

$$r = \sqrt{(x_j - x_i)^2 + (y_j - y_i)^2 + (z_j - z_i)^2} \tag{42}$$

the mutual inductance according eqn.(21) becomes:

$$M = \frac{\mu_0}{4\pi} \times \left| \left\{ \int_{-\frac{L}{2}}^{\frac{L}{2}} \int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{\left[1 + \sinh(\frac{x_i}{\alpha_k}) \sinh(\frac{x_j}{\alpha_l})\right] dx_i dx_j}{\sqrt{(x_j - x_i)^2 + (y_l'' - y_k)^2 + \left(h_l + \alpha_l \left[\cosh(\frac{x_j}{\alpha_l}) - 1\right] - h_k - \alpha_k \left[\cosh(\frac{x_i}{\alpha_k}) - 1\right]\right)^2}} + \right.\right.$$
$$\left.\left. - \int_{-\frac{L}{2}}^{\frac{L}{2}} \int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{\left[1 + \sinh(\frac{x_i}{\alpha_k}) \sinh(\frac{x_j}{\alpha_l})\right] dx_i dx_j}{\sqrt{(x_j - x_i)^2 + (y_l' - y_k)^2 + \left(h_l + \alpha_l \left[\cosh(\frac{x_j}{\alpha_l}) - 1\right] - h_k - \alpha_k \left[\cosh(\frac{x_i}{\alpha_k}) - 1\right]\right)^2}} \right\} \right| \tag{43}$$

The electromotive force induced in the loop can be obtained from the relation:

$$\mathrm{E} = -j\omega M I_k \tag{44}$$

and the loop-current becomes:

$$I_{loop} = -\frac{j\omega\mu_0 I_k}{4\pi} \times \left| \left\{ \int_{-\frac{L}{2}}^{\frac{L}{2}} \int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{\left[1 + \sinh(\frac{x_i}{\alpha_k}) \sinh(\frac{x_j}{\alpha_l})\right] dx_i dx_j}{\sqrt{(x_j - x_i)^2 + (y_l'' - y_k)^2 + \left(h_l + \alpha_l \left[\cosh(\frac{x_j}{\alpha_l}) - 1\right] - h_k - \alpha_k \left[\cosh(\frac{x_i}{\alpha_k}) - 1\right]\right)^2}} + \right.\right.$$
$$\left.\left. - \int_{-\frac{L}{2}}^{\frac{L}{2}} \int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{\left[1 + \sinh(\frac{x_i}{\alpha_k}) \sinh(\frac{x_j}{\alpha_l})\right] dx_i dx_j}{\sqrt{(x_j - x_i)^2 + (y_l' - y_k)^2 + \left(h_l + \alpha_l \left[\cosh(\frac{x_j}{\alpha_l}) - 1\right] - h_k - \alpha_k \left[\cosh(\frac{x_i}{\alpha_k}) - 1\right]\right)^2}} \right\} \right| \tag{45}$$

2.5. Mitigated magnetic field under sagging power lines using a sagging passive loop

Based on the above formulas for a single span single conductor catenary, in presence of many current sources (current carrying conductors), the superposition can be applied.

The magnitude of the magnetic flux density can be next calculated from the relationship (29), and the total *x*, *y* and *z* components of the magnetic flux density in the vicinity of the overhead sagging 3-phase power line with phase conductors (*n*), earth wires (*m*) and auxiliary conductors (*k*) forming the sagging loop can be obtained by superposition, according to the equations (30) - (32).

It should be noted, that the magnetic flux density produced by the *k–th* auxiliary current has to be obtained according to eqn. (11) - (13).

It should be pointed out, that in the calculations of the magnetic field in the observation point located at the midspan, the influence of the currents flowing in the nearby spans can be neglected. On the other hand, in the magnetic field calculations near power line towers the influence of the current in the nearest span has to be taken into account.

3. EXAMPLE

Consider the mutual inductance between a sagging conductor of a power line and a mitigation loop. The calculation has been carried out numerically for parameters: L = 400 m, H = 10 m, h = 6 m, y_i = 0 m (sagging conductor); l = 400 m, h_l = 5 m, $y_l^{'} = 1$ m, $y_l^{'''} =$ 10 m (loop). The mutual inductance according to eqn. (22) M = 22.48 μH.

A question arises how do the other line segments (nearby spans) influence the calculation result. The calculations have been carried out for three spans with parameters as above; the loop is located under the middle span. The mutual inductance in this case M = 22.56 μH. It follows from the calculation that the influence of the nearby spans on the mutual inductance can be neglected.

4. CONCLUSIONS

The paper presents procedures of determining the magnetic flux density of the field produced under sagging conductors of a power line with and without mitigation loop.

The method assumes that the line sag is described by the equation of the catenary, effects of earth currents onto magnetic field are negligible and that phase-currents have prescribed values, based on which all of the remaining currents (in earth conductors and in loop-conductors) are computed.

The mitigation effects due to the passive loop are investigated, whereas the mitigation loop can be treated as a rectangular loop (two-conductor closed mitigation loop) horizontal located under the power line or as a closed loop with two sagging conductors.

The results derived can be used as the foundation for almost every study on the magnetic fields for conditions that are almost always satisfied for power engineering applications.

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