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CONTROL OF DC MOTOR WITH A TIME DELAY CONTROLLER

Summary. The electric drive is and will probably continue to be of primary importance in engineering and technical sciences. In this work a particular issue of controlling dc motor with the time-delay controller is discussed. The time delays in the circuit emerge e.g. when the object is controlled via a great distance (space trips for instance - space ships are controlled from the Earth). The original achievement of this work is determination of the stability range of appropriate time-delay control circuit.

Keywords: DC motor, time delay controller

STEROWANIE SILNIKIEM PRĄDU STAŁEGO Z WYKORZYSTANIEM REGULATORA Z OPÓŹNIENIEM

Streszczenie. Napęd elektryczny odgrywa, i chyba będzie dalej odgrywał, fundamentalne znaczenie w technice i w naukach technicznych. W tej pracy ograniczymy się jedynie do szczególnego problemu sterowania silnikiem prądu stałego z wykorzystaniem regulatora z opóźnieniem. Opóźnienia w układzie pojawiają się np. w przypadku sterowania obiektem z dużych odległości, np. loty kosmiczne – sterownie z ziemi statkiem kosmicznym. Oryginalnym osiągnięciem w tej pracy jest wyznaczenie obszaru stabilności odpowiedniego układu regulacji z opóźnieniem.

Słowa kluczowe: Silnik prądu stałego, regulator z opóźnieniem

1. INTRODUCTION

The electric drive is and will probably continue to be of primary importance in engineering and technical sciences. The electrical machine is still a fascinating device which transforms electrical energy into mechanical energy or vice versa. There are numerous publications on modelling and control of different types of electric drive. I shall cite here two examples only: book published by team of researchers from AGH University of Science and Technology [11] in 2014 and a monograph on use of generators in national electric power system published in 2013 by scientists from Silesian University of Technology [10]. Further publications on the subject of electric drives may be found in reference lists of these books.

In the current work I will limit myself to a particular problem of controlling dc motor using a time delay controller. Delays in system occur e.g. when object is being controlled at great distance (for instance, in case of space trips, the appropriate drives of the space ship are controlled from the Earth). The trouble is that in such case even a simple model of the second order becomes a system of infinite order when the feedback loop is closed. However, in the discussed case the spectrum of closed system is a point spectrum and this makes it possible to analyze stability with procedures of finite orders.

In the previous work (and I indicate just the papers published by the employees of my home Department), different issues of controlling dc motors have been investigated. These were: LQ control [1, 2], LQR control [4], computer control [9], problem of identifying the parameters [5], problem of state reconstruction [3,8], adaptive control [4,12], speed feedback [6]. Some of the results were verified by laboratory experiments.

The current paper is arranged as follows: in the first section I shall present a simplified model of dc motor with different feedback circuits. Later I shall describe in more detail the time delay control and will conduct D-partitioning stability analysis of the closed loop. The paper is concluded by summarizing the results and pointing to the direction of further research.

2. SIMPLIFIED MOTOR MODEL AND STANDARD CONTROLLERS

Let us discuss the control circuit with separately-excited dc motor (Fig.1). Motor is controlled from the excitation side by $u(t)$ signal. If we assume that armature inductance $\boldsymbol{L}$ is nil, then armature current $\boldsymbol{I}$ is constant. The motor shaft is connected via toothed gear to the adjustable potentiometer, and its output signal is voltage $y(t)$. The angular position of the motor shaft (denoted as $x_1(t)$) determines e.g. the position of directional antenna and, at the same time, the position of the potentiometer's brush (and voltage $y(t)$ is probed at this brush). In this way the magnitude of $y(t)$ voltage determines unequivocally the angular position of directional antenna.

Next, let the angular speed of the shaft be denoted as $x_2(t) = dx_1(t)/dt$, and moment of inertia of the motor shaft be marked as $\boldsymbol{J}$. When *appropriate* procedure of simplifying the mathematical model is applied (see e.g. [2]), the control circuit shown in Fig.1 may be described with the following state equation:

$$\dot{x}(t) = Ax(t) + Bu(t), \quad y(t) = Cx(t), \quad x(t) = [x_1(t) \quad x_2(t)]^T, \tag{1}$$

where

$$A = \begin{bmatrix} 0 & 1 \\ 0 & -f \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ b \end{bmatrix}, \quad f = \frac{\mu}{J}, \quad b = \frac{k_1 k_2 I}{JR}, \quad C = [c \quad 0], \quad c \neq 0, \tag{2}$$

and k_1 and k_2 are adequate constants present in simplified (i.e. at ***L***=0 and ***I***=const) dependencies of magnetic flux on voltage *u(t)* and rotational torque on flux. If we assume that ***L*** differs from zero, then dimensions of state matrix *A* will be 3x3.

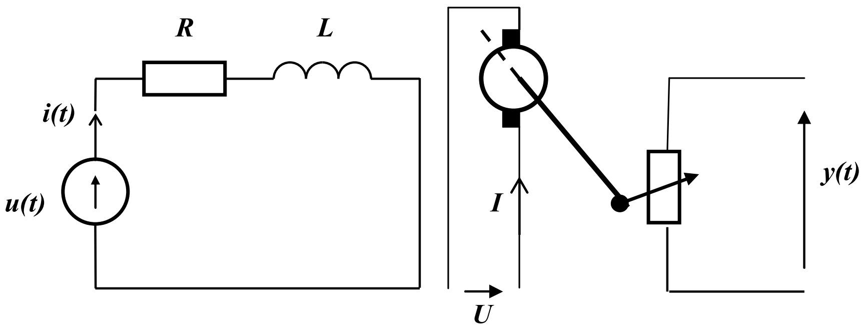

Fig.1. Control system with dc motor
Rys.1. Układ sterowania z silnikiem prądu stałego

When control circuit is designed, sometimes it is more convenient to use transmittances *G(s)* or *G(jw)* instead of model (1)-(2):

$$G(s) = C[sI - A]^{-1} B. \tag{3}$$

The standard control consists in selecting parameters of practical ***PID*** controller, characterized by following transmittance:

$$G_{PID}(s) = \frac{K_p}{T_1 s + 1} \left[1 + \frac{1}{T_i s} + \frac{T_d s}{T_2 s + 1} \right] \tag{4}$$

where T_1 and T_2 are the respective time constants determining the inertia of proportional part *P* and derivative part *D*.

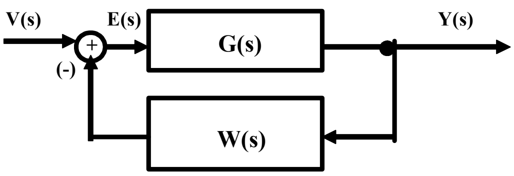

Fig. 2. Example of control circuit scheme
Rys. 2. Przykładowy schemat układu regulacji

Example of automatic control circuit structure is shown in Fig.2. In standard control we usually substitute the product of transmittances (3) and (4) and W(s) = 1 for the *G(s)*. Signal

$v(t)$ is used for setting the required value of the output $y(t)$ with Laplace transform Y(s). If the control circuit is designed properly, the error value $e(t)$ approaches zero when time approaches positive infinity. Quality control is determined by quality indicator, defined for instance as:

$$J = \int_{0}^{+\infty} e(t)^2 dt \tag{5}$$

Optimum control consists in setting controller parameters (3) in such a way, as to minimize appropriate quality indicator, e.g. the integral of the squared error (5).

Fractional control consists in using standard controllers with integrator and differentiator of fractional order. For instance

$$G_{PI^{\beta}D^{\alpha}}(s) = \frac{K_p}{T_1 s + 1}\left[1 + \frac{1}{T_i s^{\beta}} + \frac{T_d s^{\alpha}}{T_2 s + 1}\right], \quad \alpha > 0, \quad \beta > 0 \tag{6}$$

is a transmittance of practical fractional-order controller.

Control with matrix controller consists in using Ackermann's formula for determining matrix gain K in such way, as to set arbitrarily the eigenvalues of the matrix of the closed circuit $A+BK$, provided that matrix pair $(A;B)$ is controllable.

Control with dynamic feedback consists in designing an appropriate Luenberger observer and matrix controller with gain K. This control method allows practically for any modelling of the closed circuit dynamics described with matrix eigenvalues $A+BK$ and $A-GC$. Parameters K and G are selected using appropriate formulations of ***LQ*** problem (and later the solutions of appropriate two algebraic matrix Riccati equations). The wanted matrix parameters K and G exist if and only if circuit (1) is observable and may be stabilized. In the discussed case circuit (1) fulfils even stronger assumptions, since it is controllable and observable.

Computer control consists in adding to input and output a continuous-time control circuit for D/A and A/D converters (digital to analog and analog-to-digital converters; see Fig.3) and applying the control methods for discrete-time circuits.

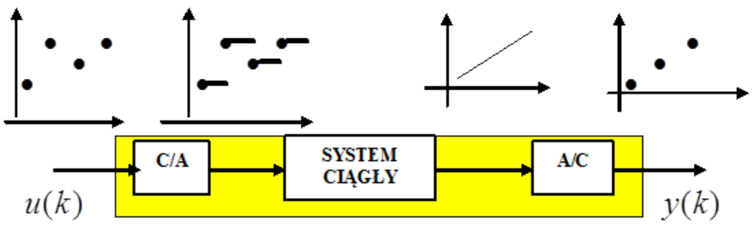

Fig. 3. Discrete-continuous circuit

Rys. 3. Układ dyskretno-ciągły

Even during synchronous operation of D/A and A/D converters a problem of selecting the operating time period is encountered. The selection of this period determines the quality of operation of entire computer control circuit [1, 2, 9].

3. TIME-DELAY CONTROLLER

Quite often the practical applications require making allowance for time delay (h>0)in information transfer. For instance, let us consider circuit (1) and the following feedback:

$$u(t) = -Ky(t-h) + v(t), \quad h > 0, \tag{7}$$

where $K>0$ is the gain coefficient of the controller, v is the quantity used for changing the position direction $y(t)$, e.g. in case of directional antenna coupled with the motor shaft (see Fig.1).

The dynamics of the closed circuits (1), (7) is characterized by characteristic quasi-equation formulated as [7, pp. 81, 82]:

$$s^2 + sf + Kbce^{-sh} = 0. \tag{8}$$

D-partitioning method was used in investigating asymptotic stability of a closed circuit. It was assumed that $bc = 1$ (see steady-state parameters of the circuit (1), (2)). The closed circuit (1), (7) is asymptotically stable, if the radices of the characteristic quasi-polynomial (8) lie in the left half-plane of the complex variable. Omitting the details, if variable $s=jw$ is substituted into (8), where w is an arbitrary real number and $j * j= -1,$ and when real part is separated from imaginary part, we obtain parametric (parameters K and $1/T1$) equation of *D*-partitioning boundaries for different time delays h>0.

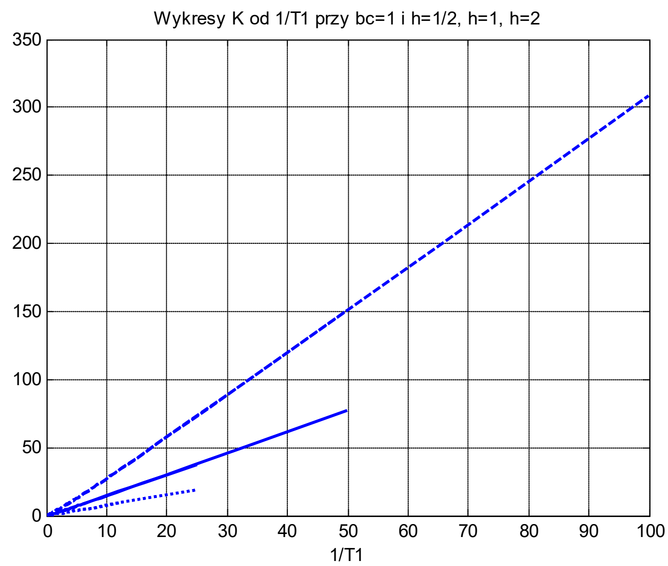

Fig.4. *D*-partitioning for different *h*
Rys.4. *D*-podział dla różnych *h*

The fragments of D-partitioning curve are shown in Fig.4, for bc = 1 and for delays h = 1/2 (upper curve), h = 1 (mid-curve), h = 2 (lower curve). The range of asymptotic stability is found beneath the graph of the function K vs. $1/T1$. The fragments of the D-partitioning curve, which are shown in Fig.4, have been selected on the basis of existing physical interpretation of the variability range of parameters given in Eq. (8).

4. CONCLUSION

A review of different design methods for dc motor control system is given in this paper. A short summary is presented of the following issues: standard procedures with practical ***PID*** controller (structure (4)), problem of optimum control with quality indicator (5), problem of selecting the settings of controller with gain matrix K, problem of selecting the matrix parameters of dynamic feedback (combination of standard controller with matrix gain K with Luenberger observer of matrix gain G), problem of computer control. Finally, analysis of stability of circuit cooperating with the controller (7) characterized by time delay h>0 has been conducted. The existence of matrix parameters K and G ensures controllability and observability of the system (1).

The original achievement of this work is analysis of stability (see Fig.4) of closed system (1), (7) with time delay. At present further research is underway, aimed at obtaining generalization for systems (1) of higher order and for different types of electric drives.

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