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RECENT PROGRESS IN THE ANALYSIS AND DESIGN OF LC TANK OSCILLATORS

Summary. In GHz wireless communication oscillators with sinusoidal current and voltages are basic sub-circuits. Already at the beginning of electronic circuits several types of oscillators with different active devices and circuit structures were developed. However, if we are restricted to integrated circuit technology for GHz applications the LC tank oscillator is an appropriate choice. In this paper a design approach is discussed in which also nonlinear specifications are included. Furthermore, a new analysis approach based on the Poincare-Andronov-Hopf theorem is discussed whereby its advantages will be illustrated by means of some examples.

Keywords: oscillator circuits, oscillator theory, oscillator design

AKTUALNE POSTĘPY W ANALIZIE I PROJEKTACH OSCYLATORÓW LC

Streszczenie. W oscyloskopach bezprzewodowych GHz z sinusoidalnym prądem i napięciem są podstawowe pod-obwody. Już na początku układów elektronicznych opracowano kilka typów oscylatorów z różnymi aktywnymi urządzeniami i strukturami obwodów. Jeśli jednak jesteśmy ograniczeni do technologii układów scalonych do zastosowań GHz, oscylator LC jest właściwym wyborem. W artykule omówiono podejście projektowe, w którym uwzględniono również specyfikacje nieliniowe. Ponadto omówione zostało nowe podejście analityczne oparte na twierdzeniu Poincare'ego-Andronova-Hopfa, dzięki któremu jego zalety zostaną zilustrowane za pomocą kilku przykładów.

Słowa kluczowe: obwody oscylatora, teoria oscylatora, projekt oscylatora

1. INTRODUCTION

At the beginning of the twentieth century, the wireless transmission of speech occurred in which a sinusoidal carrier signal was needed. First oscillator circuits for generating sinusoidal signals were developed, using electric arc and then an electronic tube. A first circuit structure for a tube oscillator used transformer coupling, but in the following capacitor and inductive coupling were introduced. The frequency determination capacitances, inductances and transformers were used, but after 1920, inductor-less oscillators and crystal oscillators appeared and other circuit structures were developed. Later on, transistors replaced the electronic tubes but the circuit structures also of oscillators survived. Even as integrated circuits were developed, electronic inductor-less or crystal oscillators dominated the circuit design. The progress of integrated technology led to integrated coils and an oscillator structure with a cross-coupled pair of transistors, capacitors and inductors were used to design a sinusoidal oscillator by Nguyen and Meyer [1] in 1992. Later on, it became well known as LC tank oscillator circuit. The cross-coupled pair of active elements goes back to a tube circuit for relaxation oscillations of Abraham and Bloch from 1919, which produces square waves instead of sinusoidal signals. For an excellent overview about the history of relaxation oscillators also from the mathematical point of view the reader is left to Ginoux and Letellier [2].

Nearly in parallel to the development of tube oscillator circuits after 1912, concepts for analysis of oscillator circuits were introduced which led to nonlinear differential equations with new properties. After first heuristic concepts the breakthrough to modern oscillator analysis was due to van der Pol, who formulated a fundamental nonlinear differential equation for transformer coupled oscillators which became known as van der Pol equation [3]. Later on, Mandelstam and Papalexi [4] and then Andronov following Poincare developed a complete theory of sinusoidal oscillators based on the bifurcation theory. Although in 1935 a complete theory of the analysis of oscillator circuits for van der Pol-like oscillators were presented by this group [5], a design concept for sinusoidal oscillators using the so-called Poincare-Andronov-Hopf theorem (PAH theorem) is missing. Using a variant of so-called Barkhausen's theorem, a necessary condition of the PAH theorem, a "linear" theory of oscillator design was applied in which only linear specifications can be taken into account. An extensive bibliography of oscillator circuits can be found in the excellent monograph of Groszkowski [6]. Only in 1979 Mees and Chua [7] presented first ideas for such a theory of oscillator design based on the PAH theorem but most of the following papers about the application of this theorem consider mainly aspects of the circuit analysis.

2. CIRCUIT EQUATION OF LC TANK OSCILLATORS

In 1986 we started with studies about a new design theory for LC oscillators using the PAH theorem. It was our goal, that also nonlinear specifications of LC oscillator circuits will be included in the design process from the beginning. The main concept was presented in a series of papers and finally summarized in the Ph.D. thesis of Keidies [11]. Later on, Mathis [13] clarified the meaning of Barkhausen's oscillatory condition by means of the Hartman-

Grobman theorem; cit. Maggio, de Feo, Kennedy [14]. After Buonomo and Schavio [15], [9] published their modelling concept of voltage controlled oscillators (VCO) we applied our design concept to LC tank bipolar and CMOS oscillators and especially VCOs. In the Ph.D. thesis of Prochaska [16] and recently in the Ph.D. thesis of Bremer [12] the complete theory and detailed application of our design concept of LC tank oscillators and VCO are contained. Essential results and previous papers are included already an overview of Mathis and Bremer [8].

Starting from the specifications of the desired oscillator or VCO, its circuit equations must be derived symbolically, whereby at least some circuit parameters should be included. For this purpose, we need circuit models with polynomial nonlinearities for the cross-coupled pair of

MOS transistors, the MOS varactor capacitors and the integrated inductors. MAPLE© is used for the rather complex symbolic calculations. Following Buonomo and Lo Schiavo [9] we derived a 2-dim. state space model of an LC tank VCO

$$\begin{pmatrix} \frac{dv_C}{dv_L} \end{pmatrix} = \begin{pmatrix} \gamma & -\frac{1}{C} \\ \frac{1}{C} & \gamma \end{pmatrix} \begin{pmatrix} v_C \\ i_L \end{pmatrix} + \begin{pmatrix} P^7(v_C, i_L) \\ 0 \end{pmatrix} \tag{1}$$

in which $\gamma \sim \left(\sigma \sqrt{I_{Bias}} \sqrt{W_n} - \frac{1}{R_t} \right)$, σ and the coefficients of the polynomial function P^7 (degree 7) are circuit parameters.

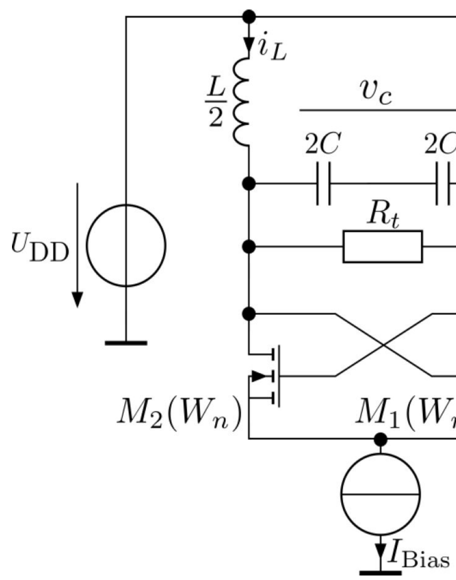

Fig. 1. LC tank MOS oscillator
Rys. 1. Oscylator MOS w zbiorniku LC

3. LC TANK OSCILLATOR DESIGN AND THE PAH THEORY

Now, it can be shown with the PAH theorem that a stable periodic solution (limit cycle) exists near the bifurcation point $\gamma = 0$. This condition can be solved exemplary with respect to the width W_n of the MOS transistors M_1 and M_2. The bifurcation point is equivalent to Barkhausen's oscillatory condition where the frequency of the oscillator is related to the eigenvalues of the matrix in eq. (i). But in addition, by means of the PAH theorem we can analyze the stability of the limit cycle. Furthermore, a symbolic variant of the average method of Krylov, Bogoliubov and Mitropolsky (KBM method) can be applied to approximate the VCO amplitude where again circuit parameters are included. In summary, we can say that the explicit circuit parameters in the approximate formulas allow the construction of a design process of LC tank oscillators and VCOs in which not only linear specifications but also non-linear specifications are contained.

Based on this analysis theory for electronic oscillators a design concept for integrated LC tank VCOs was developed; cit. Bremer [11] for further details. In *the* following, we describe the main steps of this design process:

1. The specifications of the VCO have to be defined. If it is used in a communication system the frequency, tuning range, voltage of the power supply, maximal current, phase noise and die area as well as tuning sensitivity and some additional properties must be prescribed.
2. In the next step an appropriate VCO architecture has to be chosen. However, there are only a few circuits structures within the class of LC tank MOS VCOs.
3. Now, the circuit equations have to be reduce to a 2-dimensional system of differential equations. For this purpose the circuit model of the integrated coils must be combined with other dynamical parts of the circuit such that a parallel circuit with an inductor L, a capacitor C and a nonlinear resistor R_t arise.
4. A crucial part is the construction of a suitable nonlinear circuit model for the varactors. In order to use a symbolic manipulator (e. g. MAPLE ©) we need polynomial nonlinearities for the nonlinear resistors and capacitors of the varactors. We use a least-square approximation with polynomials for the representation of the nonlinear characteristics.
5. In the next step we choose appropriate integrated coils.
6. As menioned above, our design approach is based on the PAH theorem and therefore only a few circuit parameters can be handled. Using experience of circuit design and simulation results, the number of free circuit parameters can be reduced. At the end only the bias current I_{Bias} , the transistor width of the varactor W_v and the width of the NMOS transistors W_n are the remaining parameters.
7. Now, a bifurcation analysis has to be applied and because it is a nonlinear process an iterative method is needed. Because there are three circuit parameters W_v, I_{Bias}, W_n we decided that the first two parameters will be determined by means of additional design

conditions and after chosing a starting value of W_n the iteration process is performed. If certain stopping criteria are not fulfilled the other two circuit parameters are changed and the iteration process is restarted.

8. Since the oscillator amplitude vanishes in the bifurcation point, the transistor width will be enhanced by a certain factor and we obtain a suitable design point. Now, the VCO tuning range, the oscillator amplitude, the phase noise and the higher harmonics can be approximated in the design point.
9. After an additional optimization with respect to the power a final oscillator circuit is developed which has to be improved with respect to the original specifications by means of a circuit simulator (e.g. CADENCE©).

If the VCO design is successful, the circuit can be analysed with a figure of merit for VCO which are available in the literature; cit. Bremer [12].

In Mathis and Bremer [17] the mathematical fundamentals reviewed. Furthermore, Bremer [12] discussed the design process of a example of a LC tank CMOS VCO in detail in which the specifications are determined from a 5,5GHz VCO for WLAN IEEE 802.11a application.

Some results of this thesis can be already found in Mathis and Bremer [8] where CADENCE© is used for verifying the properties of the oscillator circuit.

4. OSCILLATORS AND CARLEMAN LINEARIZATION

Although the amplitude of a limit cycle of an oscillator can be approximated by the KBM method, the original differential equations have to be transformed to polar coordinates (r,^) and we obtain rather complicate equations. Only after an averaging process we can expect polynomial differential equations for the amplitude r. Therefore, we developed a new method for approximating the limit cycle of the VCO circuit equations based on the Carleman linearization approach; cit. Weber and Mathis [10]. The main idea behind this method is that each system of polynomial differential equations can be transformed into an infinite dimensional system of linear differential equations where circuit parameters can be included.

In [10] we have shown that an infinite linear system can be approximated to a prescribed accuracy on a restricted time interval by a finite one by means of a self-consistent technique but an approximation of a limit cycle is also possible, if we use the concept of Poincare map. If the device models include transcendent nonlinearities - diode models as well as BSIM3 or EKV MOS models - we apply an additional transformation such that under weak conditions these nonlinearities can be transformed into an equivalent system with polynomial functions. This alternative approach leads to polynomial nonlinearities which is in contrast to an approximation by means of a Taylor series not only valid locally in a neighborhood of a certain point but it is valid in a global manner. The scheme of the generalized Carleman linearization with application to oscillator analysis is shown in fig. 2.

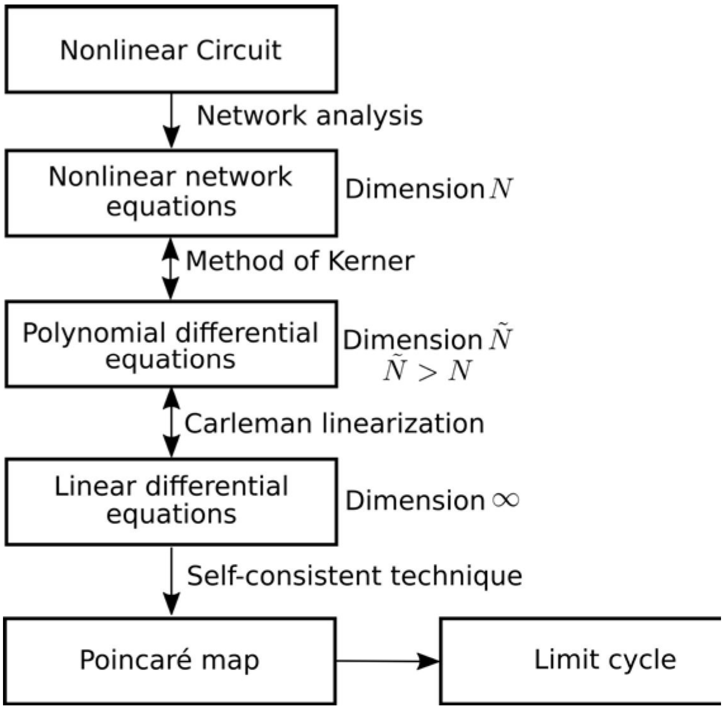

Fig. 2. Generalized Carleman linearization scheme
Rys. 2. Ogólny schemat linearyzacji Carleman

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