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ANALISIS OF FIELD DYNAMICS USING NUMERICALLY OBTAINED NORMALIZED ORDINARY MOMENTS

Summary: The article presents possibilities of describing the dynamics of the electromagnetic field making use of ordinary normalized moments. Numerical calculations has been carried out with the help of computer software COMSOL. Calculated moments have been used for determination of transfer function parameters.

Keywords: electromagnetic field, method of moments, transmittance, transfer function, field dynamics

ANALIZA DYNAMIKI PÓL Z WYKORZYSTANIEM NUMERYCZNIE WYZNACZANYCH MOMENTÓW ZNORMALIZOWANYCH ZWYKŁYCH

Streszczenie. W artykule przedstawiono możliwości opisu dynamiki pola elektromagnetycznego z wykorzystaniem momentów zwykłych znormalizowanych. W pracy wykorzystano dane otrzymane w wyniku przeprowadzenia eksperymentu numerycznego, symulacji komputerowej w programie COMSOL. Obliczone momenty posłużyły do wyznaczenia modeli transmitancyjnych badanego układu.

Słowa kluczowe: pole elektromagnetyczne, metoda momentów, transmitancja operatorowa, dynamika pola

WSTĘP

Description of dynamic systems with ordinary differential equations of appropriate order constitutes the basis for analysis of all such systems as well as for analysis of their possible interaction with other technical elements. In such cases we usually discuss ordinary differential equations of integer order n (with derivatives in relation to time) or equivalent set of n differential equations of the first order with a set of n initial conditions. This type of procedure for describing system dynamics is well known in electric circuit theory, electronics, mechanics and many other engineering branches and used for systems with so-called lumped (concentrated) parameters. It is universally used in automatics, which is a branch of science subordinate to the branches previously mentioned.

Description of dynamics in field problems (e.g. diffusion of electromagnetic field or temperature field) requires that spatial variables should be taken into account (so-called distributed parameters). Application of space discretisation methods for diffusion formula (conduction) such as e.g. FEM will result in a set of ordinary differential equations of the first order, on account of time-dependent functions, describing field in mesh nodes and with initial conditions provided.

If we assume that analysed dynamic systems are linear, both of the above seemingly different dynamic cases may be described with transfer functions; in field problems transfer functions parameters (such as time constants, delays) are functions of space.

In both cases we discuss an integer number n related to total number of meshes or nodes in circuit theory (with optional parameters R, L, C) or number of nodes in FEM mesh. This results in the fact that denominator of the transfer function (of mesh current or node potential in the electric circuit) or non-stationary field in FEM mesh will contain Laplace operator variable in L domain s of the integer and usually high power.

From the viewpoint of simplicity of analysing automatic systems, measurement transducers, electromagnetic screens etc., it is convenient to use simplified transfer function to describe system dynamics. Such transfer function contains only a few parameters. That is why equivalent models are generally used; they correspond to first order of inertia, second order with or without oscillation or m^{th} order, where integer number $m << n$ (see numerous works of A. Żuchowski, e.g. [1], or Strejc and Küpfmüller models). Selection of transfer function parameters is done in accordance with different criteria, which may result from e.g. conformity of time responses (step, impulse) or complex frequency responses of the original time function and model. Identification of parameters of simplified transfer functions for integral order of inertia, corresponding to integral order derivatives in differential equations has been researched for many years and references are abundant.

Research allowing for non-integer orders of inertia is of much more recent provenance. For instance, in field problems (with distributed parameters) we may quote such work as [2] and [3]. Contrary to diffusion problems described with parabolic equations, non-integer order of inertia used in particular in research of circuit theory issues (lumped parameters) was initially distrusted on account of uncertain physical interpretation of fractional derivatives in mesh differential equations. However, reference work on dynamics description appearing nowadays and using differential equations of non-integer order has caused the non-integer order of inertia to be adopted as a tool which describes dynamics in a more adequate way.

In current work the authors have focused on one of numerous method used in description of system dynamics, i.e. moment matching method (e.g. [4]). The aim of present research is (among other goals) to improve algorithm using moment matching method and to provide good efficiency of approximation of typical equivalent transfer functions with parameters obtained by said method. The presented technique of describing electromagnetic field dynamics will in future be applied to optimisation aiming at attainment of assumed dynamic properties of the system, which are described with several commonly used transfer functions with pre-imposed typical transfer function parameters (depending on point in space: order of inertia, time constants, delay).

1. SELECTED METHODS OF DESCRIBING DYNAMIC SYSTEMS

1.1. Transient and frequency responses

Different ways of identifying dynamic systems are used in practice. The first class groups methods which use response of the system to time-domain input function. Most popular transient responses are: step response $h(t)$ or impulse response $g(t)$.

Another input function used in testing of dynamic systems is harmonic input. This is a method using frequency domain to characterise the system. Among others, complex frequency-domain transfer function is calculated:

$$K(\mathrm{j}\omega) = \left|K(\mathrm{j}\omega)\right| e^{\mathrm{j}\alpha(\omega)} \text{ lub } K(\mathrm{j}\omega) = P(\omega) + \mathrm{j}Q(\omega) \tag{1}$$

This function enables us to draw a frequency characteristic in the complex plane.

1.2. Transfer function models

Transfer function models are important in description and modelling of properties of linear dynamic systems. They correspond to real transfer functions (either in time domain or frequency domain). There are numerous publications devoted to identification of transfer function model parameters (e.g. [1] [3] [4]). In practice identification methods for transfer function model parameters which are most commonly used are: Padé approximation, Chebyshev-Padé approximation, Routh approximation, method of continued fraction expansion and moments matching method. The last procedure is used in this paper.

1.3. Ordinary moments and ordinary normalized moments

Ordinary moment of i^{th} order is described by the relationship [1]:

$$m_i = \int_0^\infty t^i g(t)dt, \quad i = 0,1,2,..., \tag{2}$$

Normalized moment is defined by the following ratio:

$$M_i = \frac{m_i}{m_0}, \quad i = 0,1,2,..., \tag{3}$$

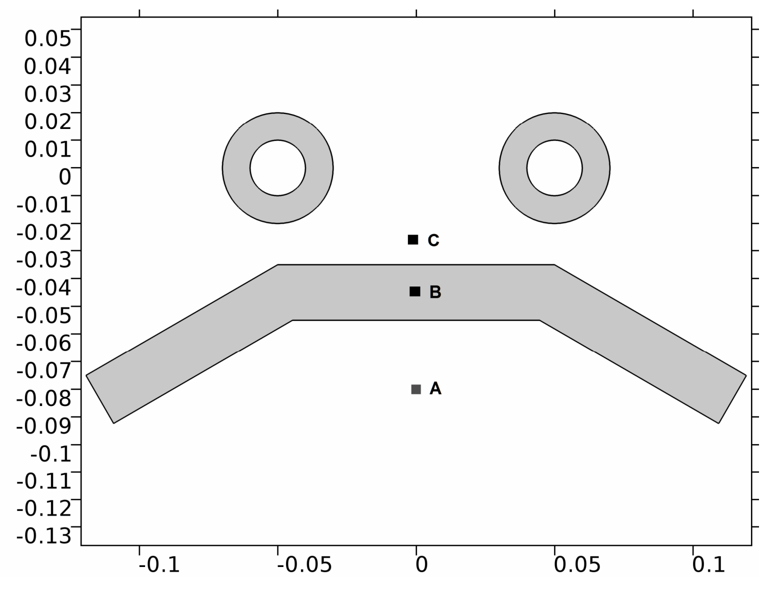
Normalized moment of first order is equal to so-called averaged time constant $\tau_z = M_1$ of the transient course $y(t)$, defined as follows [5]:

$$\tau_z = \int_0^\infty \frac{y(t) - y(t \to \infty)}{y(0) - y(t \to \infty)} dt \tag{4}$$

Advantage of the moment matching method lies in possibility of using step response $h(t)$ or impulse response $g(t)$ for determining parameters of transfer function models, obtained not only during real measurements, but also in numerical experiments. Ordinary moments may be determined quite efficiently, using complex frequency response (for relatively low frequencies). This has been described in detail in [2]. It must be pointed out that in case of systems with distributed parameters both the transient state $y(t)$ and the moments depend on spatial variables. Therefore the unknown and wanted parameters of transfer functions also depend on spatial variables. In case when field is analysed by FEM, these parameters are function of node number in the mesh used for space discretisation.

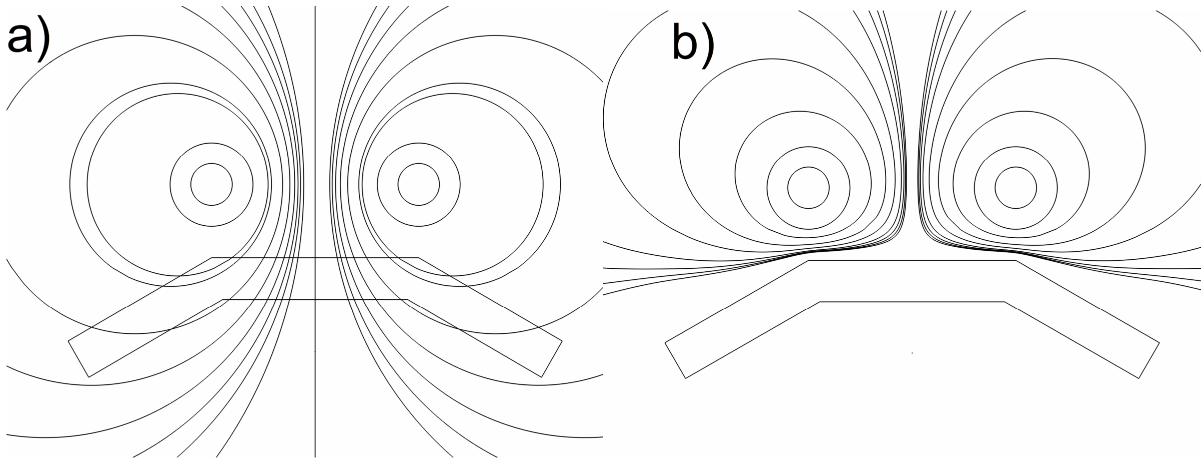
2. INVESTIGATION

Investigation of electromagnetic field dynamics has been conducted with the help of calculation model shown in Fig.1.



Rys. 1. Układ przewodów rurowych z ekranem wraz z punktami pomiarowymi
Fig. 1. Layout of pipe-type conductors with screen, location of measurement points is indicated

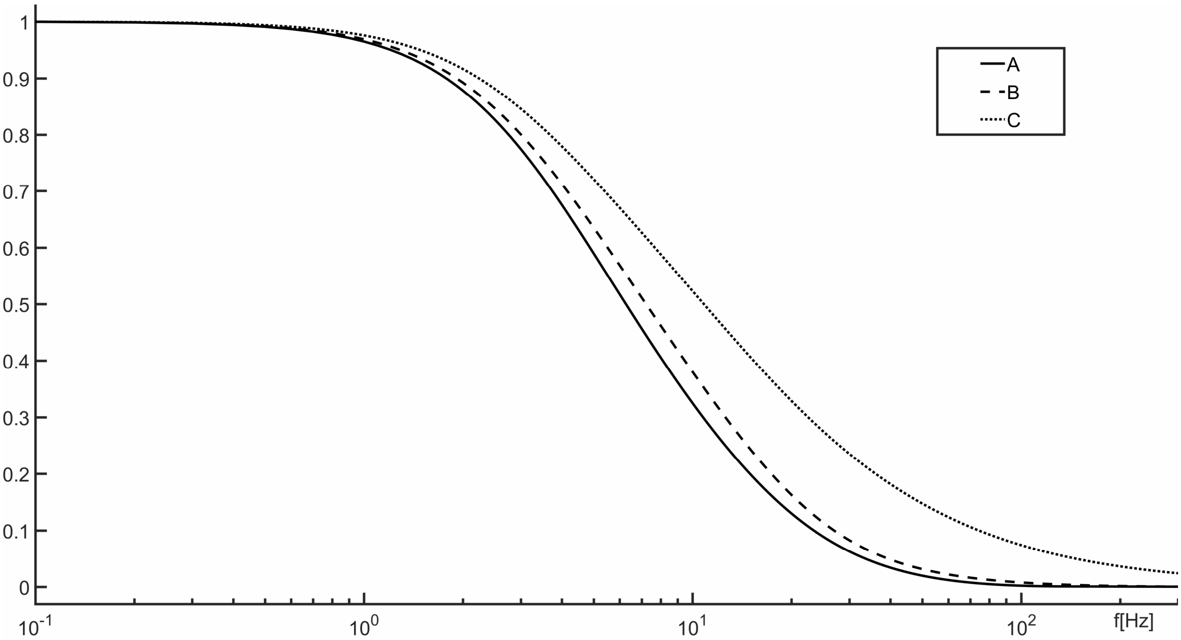
Investigated system consists of two pipe elements, which conduct currents flowing in opposite directions. Below the pipes a non-ferromagnetic conducting screen is placed. In order to verify the screen operation, analysis of field dynamics has been conducted in selected measurement points located at symmetry axis.



Rys. 2. Rozkład indukcji magnetycznej
Fig. 2. Distribution of magnetic flux density

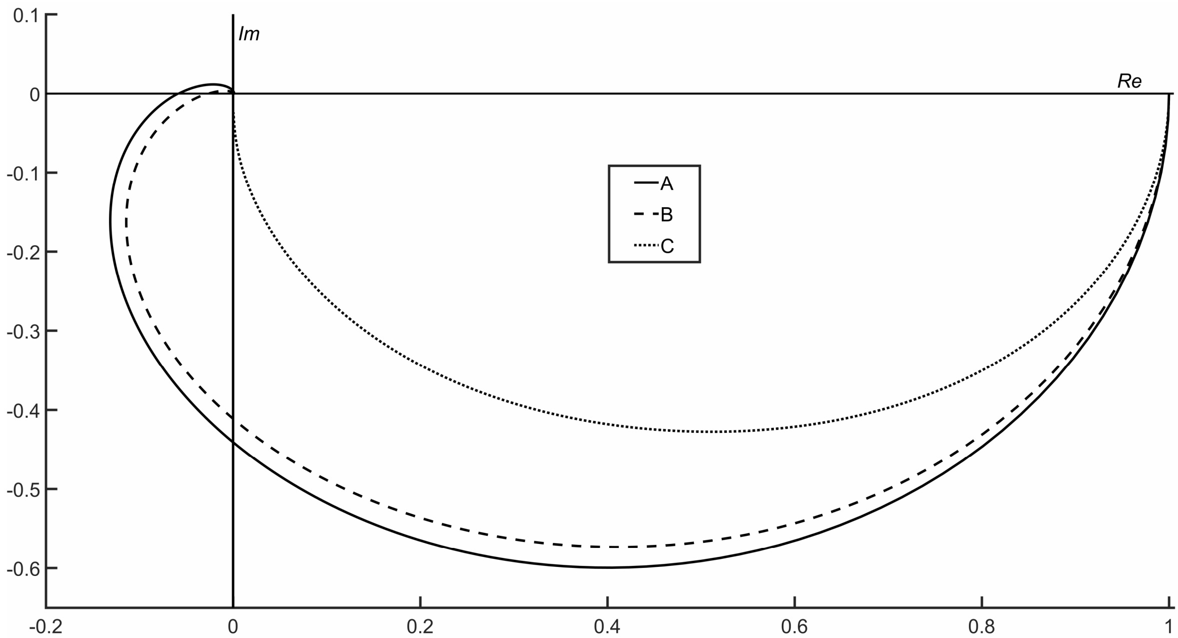
Distribution of magnetic field density is shown in Fig.2. In particular, Fig.2a demonstrates absence of screen impact for frequency $f = 0$ Hz and Fig.2b shows significant damping of field behind the screen for frequency $f = 60$ Hz.

Figs. 3 and 4 present frequency response (Fig.3) and phase response (Fig.4) of magnetic flux density in selected measurement points. The results have been obtained by solving Helmholtz equation.



Rys. 3. Charakterystyka częstotliwościowa indukcji magnetycznej w punktach pomiarowych A, B i C

Fig. 3. Frequency response of magnetic flux density at measurement points A, B and C



Rys. 4. Charakterystyka fazowa indukcji magnetycznej w punktach pomiarowych A, B i C
Fig. 4. Phase response of magnetic flux density at measurement points A, B and C

Ordinary normalized moments have been calculated using relationships given in [2].

$$M_1 = -\frac{1}{P(0)} \frac{Q(\omega_1)\omega_2^3 - Q(\omega_2)\omega_1^3}{\omega_1\omega_2^3 - \omega_2\omega_1^3} \tag{5}$$

$$M_2 = -\frac{2}{P(0)} \frac{[P(\omega_1) - P(0)]\omega_2^4 - [P(\omega_2) - P(0)]\omega_1^4}{\omega_1^2\omega_2^4 - \omega_2^2\omega_1^4} \tag{6}$$

$$M_3 = \frac{6}{P(0)} \frac{Q(\omega_2)\omega_1 - Q(\omega_1)\omega_2}{\omega_1\omega_2^3 - \omega_2\omega_1^3} \tag{7}$$

$$M_4 = \frac{24}{P(0)} \frac{[P(\omega_2) - P(0)]\omega_1^2 - [P(\omega_1) - P(0)]\omega_2^2}{\omega_1^2\omega_2^4 - \omega_2^2\omega_1^4} \tag{8}$$

Choice of pulsation $\omega_0 = 0$, ω_1 and ω_2 has been optimized using a genetic algorithm, so that condition $\omega_0 = 0 < \omega_1 < \omega_2$ should be fulfilled. The objective function was defined as minimization of difference between step response $h_z(t)$ calculated for simplified equivalent transfer function models and step response $h(t)$ calculated by solving current conduction equation with the help of COMSOL software:

$$\int_0^{\omega_g} \left(h_z(t) - h(t)\right)^2 d\omega = \min \tag{9}$$

Calculated moments M_1 do M_4 have been applied to determination of transfer function parameters in accordance with formulas given in Table 1. Adoption of criterion (9) in the process of optimising selection of ω_1 and ω_2 pulsation significantly improves determination of the moments in relation to criterion of minimum mean-square error of equivalent harmonic transfer function as implemented in [2]. In the last case the application of moment method has proved to be less effective for approximation over a wide range of mesh node numbers. For instance, in case of points located in front of the screen, some of time transmittance parameters have assumed negative values. This fact has been recognized in publications on the subject (e.g. [1]) and proves only that not all integral transforms (which represent some approximations of real system dynamics) may be used in each case (in any arbitrary mesh node). Application of optimisation for selection of frequency (Eqs. (5) to (8)) based on criterion of minimum mean-square error for step responses (9) has produced much better results. Negative values of time parameters have appeared for measurement point C (located in front of the screen), where step response does not exhibit this typical delay in the initial phase for models h2 and h3 only.

Table 1

Dependence of equivalent transfer function parameters on moments [3]

Symbol	**Model**	**Model parameters**	**Relationship to τ_z**
h1	$\frac{k_0}{1+s\tau_z}$	$\tau_z = M_1$	
h2	$\frac{k_0 \exp(-s\tau_0)}{1+s\tau}$	$\tau = \sqrt{M_2 - M_1^2}$ $\tau_0 = M_1 - \sqrt{M_2 - M_1^2}$	$\tau_z = \tau_0 + \tau$
h3	$\frac{k_0}{(1+s\tau_1)(1+s\tau_2)}$	$\tau_1 = \frac{1}{2}\left(M_1 - \sqrt{2M_2 - 3M_1^2}\right)$ $\tau_2 = \frac{1}{2}\left(M_1 + \sqrt{2M_2 - 3M_1^2}\right)$	$\tau_z = \tau_1 + \tau_2$
h4	$\frac{k_0}{(1+s\tau_m)^m}$	$m = \frac{M_1^2}{M_2 - M_1^2}, \quad \tau_m = \frac{M_1}{m}$	$\tau_z = m\tau_m$
h5	$\frac{k_0 \exp(-s\tau_0)}{(1+s\tau_m)^m}$	$m = \frac{4(M_2 - M_1^2)^3}{(M_3 - 3M_1M_2 + 2M_1^3)^2}$ $\tau_m = \sqrt{\frac{M_2 - M_1^2}{m}}$ $\tau_0 = M_1 - \sqrt{m(M_2 - M_1^2)}$	$\tau_z = \tau_0 + m\tau_m$

Parameters of equivalent transfer function models have been determined on the basis of formulas (5) to (8) and Table 1. The data, time constants and delays are shown in Table 2.

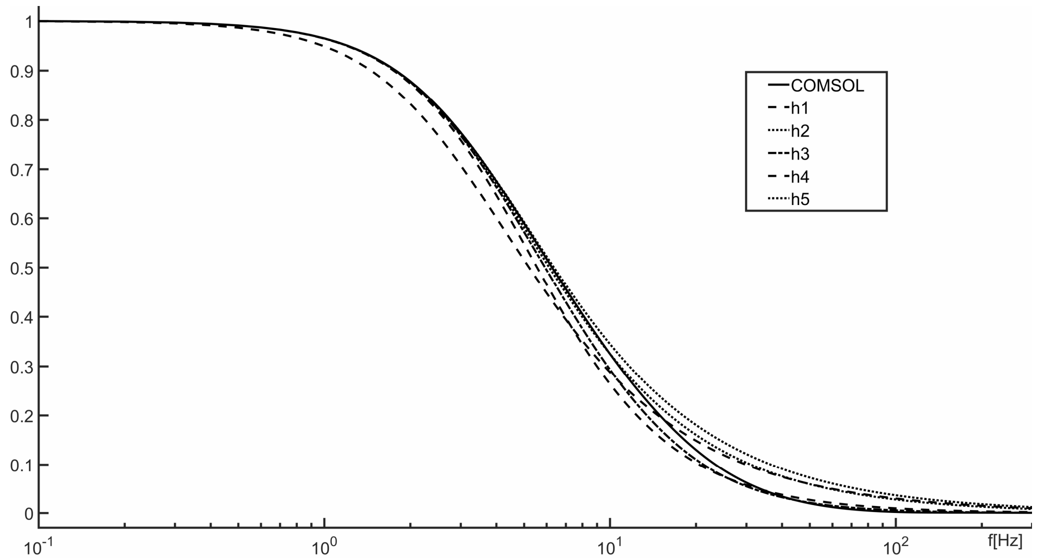
Table 2

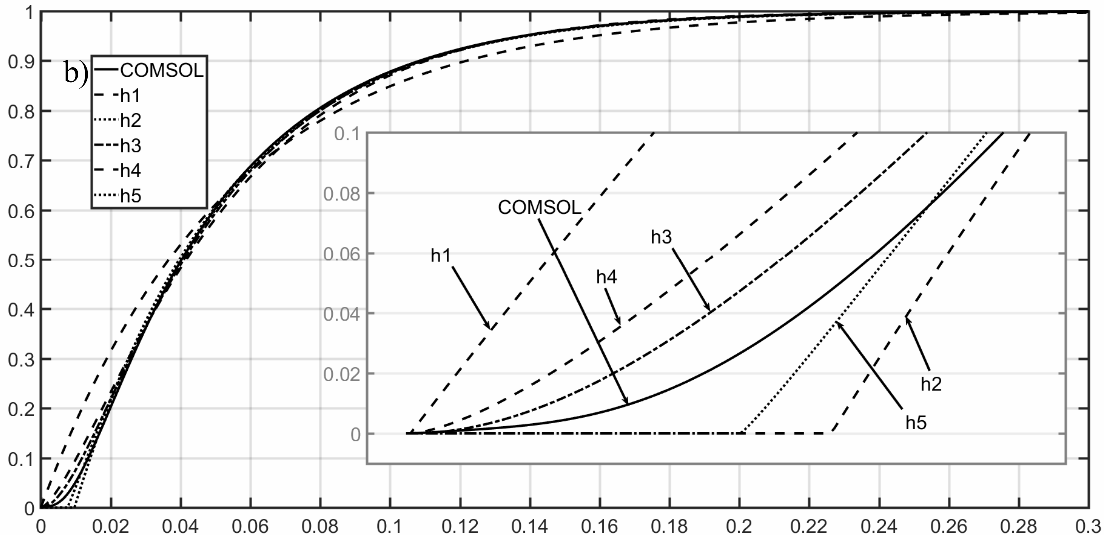
Parameters of transfer function models calculated for three specified measurement points

Symbol shown in drawings	Model and its parameters		Measurement point		
			A	B	C
h1	$\frac{k_0}{1+s\tau_z}$	τ_z	0,0530	0,0461	0,0274
h2	$\frac{k_0 \exp(-s\tau_0)}{1+s\tau}$	τ	0,0433	0,0407	~~0,0359~~
		τ_0	0,0096	0,0055	~~-0,0069~~
h3	$\frac{k_0}{(1+s\tau_1)(1+s\tau_2)}$	τ_1	0,0108	0,0059	~~-0,0102~~
		τ_2	0,0420	0,0403	~~0,0253~~
h4	$\frac{k_0}{(1+s\tau_m)^m}$	τ_m	0,0355	0,0359	0,0442
		m	1,4915	1,2875	0,6629
h5	$\frac{k_0 \exp(-s\tau_0)}{(1+s\tau_m)^m}$	τ_0	0,0076	0,0064	0,0016
		τ_m	0,0413	0,0417	0,0468
		m	1,0981	0,9536	0,5932

Comparison of frequency amplitude responses obtained with simulations run with COMSOL software with curves calculated for transfer function models and derived from normalized moments given in Table 2 is shown in Figs. 5a, 6a and 7a. Comparison of step responses obtained with simulations run with COMSOL software with step responses for transfer function models given in Table 2 is shown in Figs. 5b, 6b and 7b.

a)

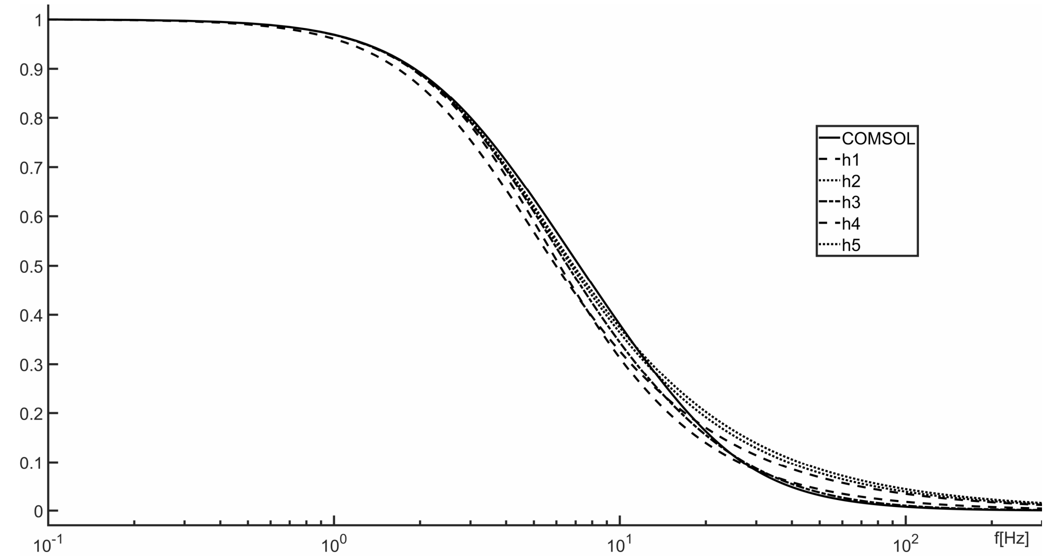


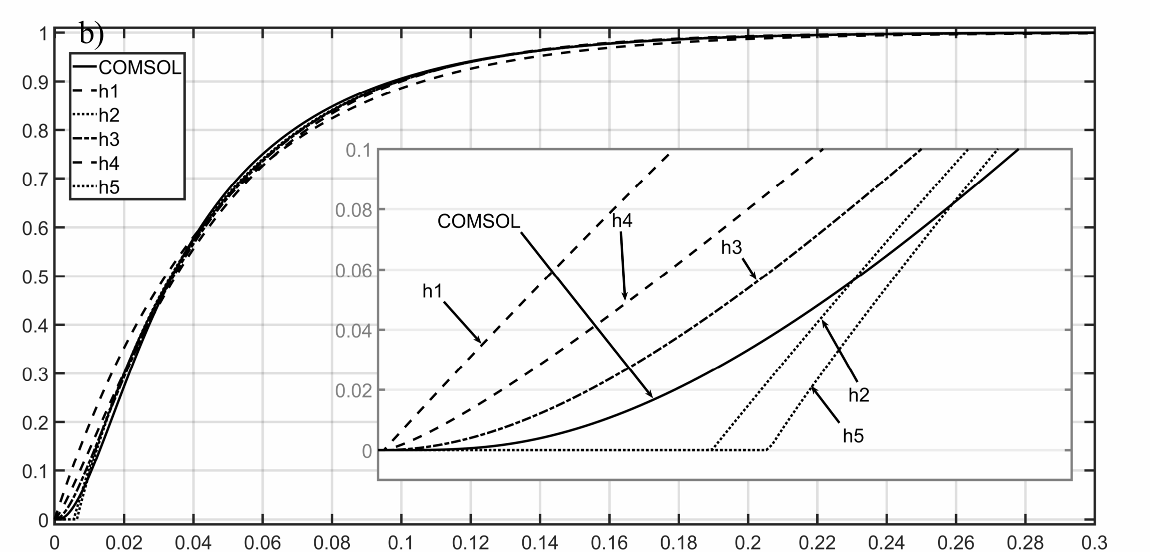


Rys. 5. Porównanie charakterystyki częstotliwościowej amplitudowej a) i odpowiedzi skokowej b) w punkcie pomiarowym A
Fig. 5. Comparison of: a) amplitude frequency response and b) step response at measurement point A

Charts presented in Figs. 5 and 6 confirm good approximation efficiency of selected simplified transfer function models for measurement point A (located behind the screen) and measurement point B (located in the screen). The greatest (and expected) divergence occurs in the initial phase of the curve for step response (see magnification in Figs. 5b and 6b).

a)

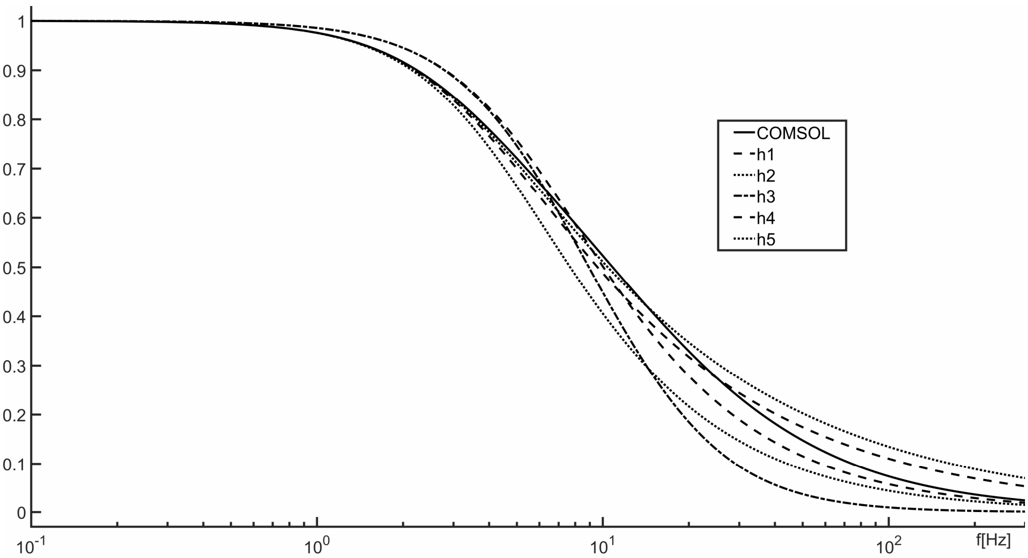


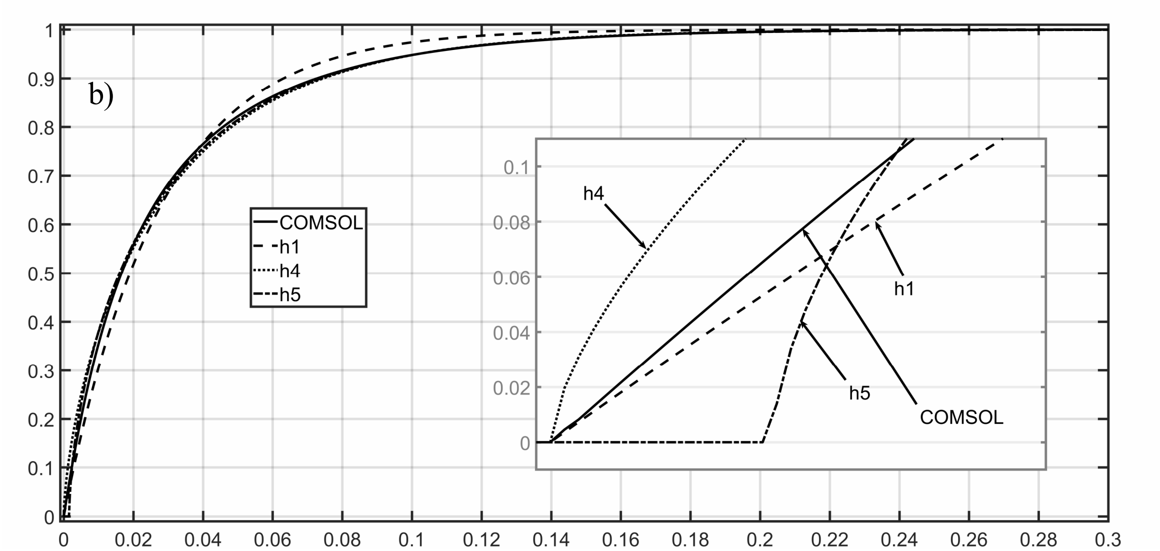


Rys. 6. Porównanie charakterystyki częstotliwościowej amplitudowej a) i odpowiedzi skokowej b) w punkcie pomiarowym B
Fig. 6. Comparison of: a) amplitude frequency response and b) step response at measurement point B

Analysing Fig.7a (amplitude frequency response), we may observe that models of equivalent transfer functions obtained from calculated normalized moments are inferior. Fig.7b contains also three out of five of step responses of adopted transfer function models. This is due to the fact that in case of models denoted as h2 and h3, the calculated time constants were negative (*cf.* Table 2).

a)





Rys. 7. Porównanie charakterystyki częstotliwościowej amplitudowej a) i odpowiedzi skokowej b) w punkcie pomiarowym C
Fig. 7. Comparison of: a) amplitude frequency response and b) step response at measurement point C

3. CONCLUSION

The conducted numerical experiments have shown that adopted transfer function models (see Table 1 and 2) are characterised by high efficiency of approximation in measurement points A (behind the screen) and B (at the screen). This is confirmed by time-domain and frequency-domain curves shown in Figs. 5 and 6. However, not all of the equivalent transfer function models may be applied to measurement point C (located in front of the screen). This is due to the fact, that typical diffusion of electromagnetic field does not occur at this point, and transient state is a "side effect" of interaction of the screen and (in part) eddy-current effects in pipe conductors. It must be noted that calculated order of inertia is much lower than 1. This might be expected from analysing the course of frequency curve in the complex plane ("flattened" curve C) shown in Fig.4.

Numerical experiments have proved that better results and better approximation efficiency is provided by selection of two frequencies ω_1 and ω_2 (genetic algorithms have been applied in optimisation) used for interpolation of real and imaginary parts of frequency response. Next, four modified moments are determined, using a mean-square error criterion for step responses of the model and original function. This procedure is superior to the criterion adopted for amplitudes of identical frequency responses and suggested in [2].

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