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APPROXIMATE BEM ANALYSIS OF THIN ELECTROMAGNETIC SHIELD

Summary. A method of approximate analysis of a thin electromagnetic shield is considered and proposed in the paper. Due to presumably small thickness of the shield, its numerical analysis is troublesome. Applying the Boundary Element Method (BEM) to solve equations for a thin shield creates two difficulties: significant increase of the number of algebraic equations, and the presence of nearly singular integrals. The proposed model avoids them both by using an approximate analytical solution for the shield. Numerical examples confirm its usability.

Keywords: BEM, thin shells, electromagnetic shielding, Helmholtz equation

1. INTRODUCTION

Some sort of equipment needs electromagnetic (EM) shielding, which can be achieved by placing it in a conductive shell (enclosure). Analysis of EM field in such a configuration requires solving the field equations at least in three regions. Besides, geometrical complexity of the problem involves numerical methods. One of such methods can be BEM (boundary element method) [1, 2, 6, 10, 13], especially, if the exterior extends considerably. However, if
the shield is relatively thin, the problem of suitable discretization of its surface appears. In addition, some BEM integrals become nearly singular, what requires specific treatment and enlarges the overall computational effort. For that reason, such thin shells should be treated in a special way. For example, thin shells have been considered in [3-5, 7-11], to recall a few only. This paper presents an approximate method of reducing the BEM equations arising in EM shielding analysis. For simplicity, the considerations are limited to 2D problems.

2. PROBLEM DESCRIPTION AND BEM ANALYSIS

2.1. Problem description and governing equations

An EM shield, \( \Omega_1 \), is placed in free space, \( \Omega_0 \), and encloses the protected region, \( \Omega_2 \) – Fig. 1. The external and internal surfaces of the shield are referred to as \( S_1 \) and \( S_2 \), respectively. The shield is considered to be very thin of constant thickness \( d \), relative permeability \( \mu_{r1} = \text{const} \), and electric conductivity \( \gamma_1 = \text{const} \), while \( \mu = \mu_0 \) and \( \gamma = 0 \) in the protected region and the free space. Such a configuration is affected by an external transverse harmonic magnetic field, \( B_s \), of angular frequency \( \omega \).

\[ B_s, \omega \]

\[ \mu_0, \gamma = 0 \]

\[ \mu_{r1}, \gamma_1 \]

\[ S \]

\[ \mu_0, \gamma = 0 \]

\[ y \]

\[ x \]

\[ z \]

Fig. 1. Problem description
Rys. 1. Konfiguracja obliczeniowa

In 2D problems the vector magnetic potential \( \mathbf{A} \) (\( \mathbf{B} = \nabla \times \mathbf{A} \)) can be chosen to have only a \( z \)-component. Time harmonic dependency of excitation \( \mathbf{B}_s \) permits using the phasor notation. The phasor of the \( z \)-component of vector magnetic potential \( \mathbf{A} \) fulfills the following equations in particular regions [12, 14]:

\[ \begin{cases} \nabla^2 A^{(m)} = 0 & \text{for } \Omega_m, m = 0, 2, \\ \nabla^2 A^{(1)} - \kappa^2 A^{(1)} = 0 & \text{for } \Omega_1, \end{cases} \tag{1} \]

where

\[ \kappa^2 = j \omega \mu_{r1} \mu_0 \gamma_1, \tag{2} \]

and \( j \) is the imaginary unit. Field continuity conditions on boundaries \( S_1 \) and \( S_2 \) are as follows:
\[ A^{(0)}|_{S_1} = A^{(1)}|_{S_1}, \quad A^{(1)}|_{S_2} = A^{(2)}|_{S_2}. \]  
(3)

\[ \frac{\partial A^{(0)}}{\partial n}|_{S_1} = -\frac{1}{\mu_1} \frac{\partial A^{(1)}}{\partial n}|_{S_1}, \quad \frac{\partial A^{(2)}}{\partial n}|_{S_2} = -\frac{1}{\mu_1} \frac{\partial A^{(1)}}{\partial n}|_{S_2}. \]  
(4)

Far from the shield, theoretically – in the infinity, where the influence of the shield, the vector magnetic potential tends to the vector magnetic potential of externally applied magnetic field:

\[ A|_{\infty} \to A_s, \]  
(5)

where \( A_s \) is chosen so that

\[ B_s = \nabla \times (A_s I_c). \]  
(6)

### 2.2. Standard BEM model

The standard BEM applied to this problem leads to the following system of equations:

\[
\begin{align*}
\mathbf{H}_1^{(0)} A_1^{(0)} &= \mathbf{G}_1^{(0)} Q_1^{(0)} + A_s \\
\mathbf{H}_1^{(1)} A_1^{(1)} + \mathbf{H}_2^{(1)} A_2^{(1)} &= \mathbf{G}_1^{(1)} Q_1^{(1)} + \mathbf{G}_2^{(1)} Q_2^{(1)} \\
\mathbf{H}_2^{(2)} A_2^{(2)} &= \mathbf{G}_2^{(2)} Q_2^{(2)}
\end{align*}
\]  
(7)

Vectors \( \mathbf{A}_l^{(m)} \) and \( \mathbf{Q}_l^{(m)} \) contain nodal values of \( \mathbf{A}^{(m)} \) and \( \partial_n \mathbf{A}^{(m)} \) on boundary \( S_l \). Vector \( \mathbf{A}_s \) contain nodal values of potential \( \mathbf{A}_s \) on boundary \( S_1 \). Matrices \( \mathbf{G}^{(m)} \) and \( \mathbf{H}^{(m)} \) are built from boundary integrals \( g_{ijk}^{(m)} \) and \( h_{ijk}^{(m)} \), respectively, where

\[
g_{ijk}^{(m)} = \int_{S_j} N_k G^{(m)} \, dS, \quad h_{ijk}^{(m)} = \int_{S_j} N_k \frac{\partial G^{(m)}}{\partial n} \, dS, \]  
(8)

where \( N_k \) are shape functions used for approximation of \( \mathbf{A} \) and \( \partial_n \mathbf{A} \) in boundary elements, and \( G^{(m)} \) is the fundamental solution of equation for domain \( \Omega_m \). The fundamental solution depends on the distance, \( R \), between arbitrary point \( i \) and point \( P \) lying on boundary \( S \). It equals

\[ G(R) = \frac{1}{2\pi} \ln \frac{1}{R} \]  
(9)

for \( \kappa = 0 \) (domains \( \Omega_0 \) and \( \Omega_2 \), and
for $\kappa \neq 0$ (domain $\Omega_1$), where $K_0$ is the modified Bessel function of the second kind of order 0. Detailed construction of matrices $G^{(m)}$ and $H^{(m)}$ is presented in [2, 6, 10].

Boundary conditions (3) and (4) rewritten in discrete form are as follows:

$$A_1^{(0)} = A_1^{(1)} = A_1, \quad A_2^{(1)} = A_2^{(2)} = A_2,$$  \hspace{1cm} (11)

$$Q_1^{(0)} = -\beta Q_1^{(1)}, \quad Q_2^{(2)} = -\beta Q_2^{(1)},$$ \hspace{1cm} (12)

where

$$\beta = \frac{1}{\mu_{t1}}.$$ \hspace{1cm} (13)

The final equations can be written as

$$\begin{bmatrix} H_1^{(0)} & \beta G_1^{(0)} & 0 & 0 \\ H_1^{(1)} & -G_1^{(1)} & H_2^{(1)} & -G_2^{(1)} \\ 0 & 0 & H_2^{(2)} & \beta G_2^{(2)} \end{bmatrix} \begin{bmatrix} A_1^{(0)} \\ A_1^{(1)} \\ A_2^{(1)} \\ A_2^{(2)} \end{bmatrix} = \begin{bmatrix} A_1 \\ Q_1^{(1)} \\ A_2 \\ Q_2^{(2)} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$ \hspace{1cm} (14)

The system of equations, from now on referred to as the standard BEM model, is mathematically correct for any values of parameters (such as $d$, $\beta$, $A_s$, boundary shapes). However, numerical tests show that small values of $d$ can be troublesome. This is because the integrands in integrals (8) can have very sharp peaks (but still finite). Such integrals are called nearly singular, by analogy to singular integrals, whose integrands have infinite peaks. Numerical evaluation of nearly singular integrals require a considerable computational effort, and even then can lead to significant numerical errors. Moreover, since corresponding nodes on $S_1$ and $S_2$ become very close, the BEM equations for them are almost linearly dependent. These two disadvantages lead to troubles during numerical computations.

### 2.3. Approximate BEM model

There are some methods of avoiding the aforementioned disadvantages [3-5, 7-11]. The one presented here consist in using a semi-analytical solution in the thin shield. The first idea that comes on mind is similar to presented in [3, 5], and consists in expanding the solution in the shell into power series. As a result, the values of $A$ and $\partial_n A$ at the corresponding points lying on boundaries $S_1$ and $S_2$ are connected with the following approximate relationship:
\[ Q_1^{(1)} = \left. \frac{\partial A_1}{\partial n} \right|_{S_1} \approx \frac{A_1 - A_2}{d}. \] (15)

Unfortunately, this implies \( Q_2^{(1)} = -Q_1^{(1)} \), and no information on value of \( \kappa \) can be introduced into the equations. Therefore, this approach must be rejected, and a method of taking into account \( \kappa \) must be found.

To achieve this, observe that if the shield is thin enough and the BEM discretization is fine enough, the shell between two corresponding boundary elements lying on \( S_1 \) and \( S_2 \) may be approximately regarded as a fragment of infinite plate. In such a plate the general solution of the second of Eqs. (1) for \( \kappa \neq 0 \) can be expressed as

\[ A(x) = C_1 \cosh \kappa x + C_2 \sinh \kappa x, \] (16)

where \( C_1 \) and \( C_2 \) are constants. Assuming that \( A(0) = A_1 \) and \( A(d) = A_2 \), one can eliminate the constants and obtain

\[ A(x) = \frac{A_1 \sinh \kappa (d - x) + A_2 \sinh \kappa x}{\sinh \kappa d}, \] (17)

and consequently,

\[ \frac{\partial A}{\partial x} = \kappa \frac{-A_1 \cosh \kappa (d - x) + A_2 \cosh \kappa x}{\sinh \kappa d}. \] (18)

Therefore, the normal derivatives of \( A \) for \( x = 0 \) and \( d \), can be expressed as

\[ Q_1 = -\left. \frac{\partial A}{\partial x} \right|_0 = \sigma A_1 - \tau A_2, \] (19)

\[ Q_2 = -\left. \frac{\partial A}{\partial x} \right|_d = -\tau A_1 + \sigma A_2, \] (20)

where

\[ \sigma = \frac{\kappa \cosh \kappa d}{\sinh \kappa d}, \quad \tau = \frac{\kappa}{\sinh \kappa d}. \] (21)

Applying Eqs. (19) and (20) to the thin shield yields the following approximate relationships:

\[ Q_1^{(1)} \approx \sigma A_1 - \tau A_2, \quad Q_2^{(1)} \approx -\tau A_1 + \sigma A_2. \] (22)

It is worth noting that the approach described in [3, 5] can be obtained exactly in the same way. In fact, Eqs. (21) for \( \kappa \to 0 \) give
\[
\lim_{\kappa \to 0} \sigma = \lim_{\kappa \to 0} \tau = \frac{1}{d},
\]

and relationship (19) becomes identical with (15).

Using Eqs. (22) in (14) allows eliminating the BEM equations for the shield. The resulting equations are as follows

\[
\begin{bmatrix}
H_1^{(0)} + \sigma \beta G_1^{(0)} & -\tau \beta G_1^{(2)} \\
-\tau \beta G_2^{(2)} & H_2^{(2)} + \sigma \beta G_2^{(2)}
\end{bmatrix}
\begin{bmatrix}
A_1 \\
A_2
\end{bmatrix} = \begin{bmatrix}
A_s \\
0
\end{bmatrix}.
\]

In the subsequent paragraphs, the system of equations is called the approximate BEM model (ABEM). It has half the number of equations given by the standard BEM model (14) and no nearly singular integrals occur in it (for sufficiently regular boundary). However, one should keep in mind that this model is a result of assumption that the shell can be locally treated as planar. In fact, this not always may be acceptable, but there are situations in which it should work well.

3. NUMERICAL RESULTS

3.1. General remarks

Both models were implemented in Mathematica 7.0 and tested in various conditions. The considered shields were long cylinders of very thin walls and constant cross-section. It was assumed that the externally applied magnetic field \(B_s\) was uniform and had the following form

\[
B_s = B_0 I_x, \quad A_s = B_0 y.
\]

Two kinds of boundary elements were used in numerical tests: constant (with one node in the midpoint) and quadratic (with three nodes in the beginning, midpoint and endpoint). This refers, however, only to the field approximation (\(A\) and \(\partial n A\)), since the geometry was always approximated with quadratic curves. This allowed taking into account the shape of boundary quite precisely. The same set of nodes was used for both kinds of elements.

Equation systems (14) and (24) were solved with use of Mathematica’s built-in routines (LinearSolve). Integrals (8) were evaluated as follows:

- for Laplace equation (domains \(\Omega_0\) and \(\Omega_2\): analytical integration for straight geometry, special treatment with use of logarithmic Gaussian quadrature for singular cases and curvilinear geometry, numerical integration according to the four-zoned scheme described thoroughly in [3],
for Helmholtz equation (domain $\Omega_1$, only the standard BEM): special treatment with singularity exclusion for singular cases, numerical integration based on the four-zoned scheme for non-singular cases.

The four-zoned scheme introduces four zones, whose limits are determined by the endpoints of a boundary element and three parameters $0 < s_1 < s_2 < s_3 < \infty$. Depending on the zone in which point $i$ is located, different integration method or quadrature order is used, in accordance to the rule: “the closer the point to the boundary element the more refined integration”. In the nearest zone, the Mathematica’s built-in function (NIntegrate) is used to gain appropriate accuracy with minimal programming effort, and in the other zones – the Gaussian quadrature of orders $GQO_1 > GQO_2 > GQO_3 > 0$ (to decrease computation time). Plots show the actual values of the integration parameters as “$s_1; GQO_1 > s_2; GQO_2 > s_3; GQO_3$”.

3.2. Cylindrical shield

The first benchmark problem was a cylindrical shield of circular cross-section (Fig. 2), whose internal and external radii were $R_2$ and $R_1 = R_2 + d$, respectively. It was used to check the validity of both models, because it has an analytical solution, which is as follows:

$$A^{(0)} = B_0R_1 \left(1 + \frac{v - w}{v + w} \frac{R_1^2}{r^2} \right) \frac{r}{R_1} \sin \varphi,$$

(26)

$$A^{(1)} = B_0R_1 \frac{2}{v + w} \left[pI_1(\kappa r) - qK_1(\kappa r)\right] \sin \varphi,$$

(27)

$$A^{(2)} = B_0R_1 \frac{2\beta(1 + \delta)}{v + w} \frac{r}{R_1} \sin \varphi,$$

(28)

where $I_1(z)$ and $K_1(z)$ are modified Bessel functions of the first and second kind of order one, respectively, $\beta$ is given by Eq. (13), $\delta$ is the relative thickness of the shell defined as

$$\delta = \frac{d}{R_2},$$

(29)

and

$$p = K_1(\kappa R_2) - \beta \kappa R_2 K'_1(\kappa R_2),$$

(30)

$$q = I_1(\kappa R_2) - \beta \kappa R_2 I'_1(\kappa R_2),$$

(31)

$$v = pI_1(\kappa R_1) - qK_1(\kappa R_1),$$

(32)

$$w = \beta \kappa R_1[pI'_1(\kappa R_1) - qK'_1(\kappa R_1)].$$

(33)
The approximate BEM model uses relationships (22). It is worth to investigate if they are applicable for cylindrical shield. To find it out it is necessary to evaluate the equivalents of coefficients $\sigma$ and $\tau$ and compare them with expressions (21). From Eqs. (26) and (28) through continuity conditions (3) one obtains, respectively

$$A_1 = A^{(0)} \bigg|_{r=R_1} = B_0 R_1 \frac{2v}{v+w} \sin \varphi,$$

$$A_2 = A^{(2)} \bigg|_{r=R_2} = B_0 R_1 \frac{2\beta}{v+w} \sin \varphi.$$  

The same equations with continuity conditions (4) lead to expressions for the normal derivatives on $S_1$ and $S_2$:

$$Q_1^{(1)} = \frac{\partial A^{(1)}}{\partial r} \bigg|_{r=R_1} = \frac{1}{\beta} \frac{\partial A^{(0)}}{\partial r} \bigg|_{r=R_1} = B_0 \frac{2w}{v+w} \sin \varphi,$$

$$Q_2^{(1)} = -\frac{\partial A^{(1)}}{\partial r} \bigg|_{r=R_2} = \frac{1}{\beta} \frac{\partial A^{(2)}}{\partial r} \bigg|_{r=R_2} = -B_0 \frac{2(1+\delta)}{v+w} \sin \varphi.$$  

By requiring to satisfy the following relationships:

$$\begin{cases} Q_1^{(1)} = \sigma' A_1 - \tau' A_2, \\ Q_2^{(1)} = -\tau' A_1 + \sigma' A_2, \end{cases}$$

one finds that

$$\sigma' = \frac{1}{\beta R_1} \frac{w^2 + (1+\delta) \beta^2}{v^2 - \beta^2}, \quad \tau' = \frac{1}{R_1} \frac{v(1+\delta) + w}{v^2 - \beta^2}. \quad (34)$$

These expressions for small $\delta$ should be compared with those given by Eqs. (21). It is convenient to introduce the dimensionless parameter $\kappa$ defined as...
Since $\kappa R_1 = \kappa R_2 (1 + \delta) = \kappa + \kappa \delta$, quantities (30)-(33), and consequently (34), can be expressed in terms of dimensionless parameters $\kappa$, $\delta$ and $\beta$. The goal is to consider their approximate values for small $\delta$. This is considered separately for $|\kappa \delta| << 1$ and $|\kappa \delta| \geq O(1)$. If $\Delta$ is the skin depth for the shield, it holds

$$\kappa \delta = \kappa d = \frac{d}{\Delta} (1 + j).$$

Thus, case $|\kappa \delta| << 1$ is equivalent to $d << \Delta$, what corresponds to a small attenuation in the shield, while $|\kappa \delta| \geq O(1)$ is equivalent to $d \geq O(\Delta)$ – a relatively large attenuation.

Beginning with the last case, observe that it implies $|\kappa| >> 1$ (since $\delta << 1$). Therefore, using the following asymptotic expansions for $|z| >> 1$:

$$I_1(z) \approx I'_1(z) \approx \frac{1}{\sqrt{2\pi z}} e^z, \quad K_1(z) \approx -K'_1(z) \approx \frac{\pi}{\sqrt{2z}} e^{-z},$$

it is easy to show that

$$v \approx \frac{\sinh \kappa \delta + \beta \kappa \cosh \kappa \delta}{\kappa}, \quad w \approx \beta (\cosh \kappa \delta + \beta \kappa \sinh \kappa \delta),$$

and consequently, after some transformations

$$\sigma' \approx \kappa \frac{\cosh \kappa d}{\sinh \kappa d} = \sigma, \quad \tau' \approx \kappa \frac{1}{\sinh \kappa d} = \tau.$$

It is worth noting that $\beta$ does not matter at all in this case.

If $|\kappa \delta| << 1$, we can use expansions into power series of $\kappa \delta$. Using various identities for Bessel functions, like

$$I'_v(z)K_v(z) - I_v(z)K'_v(z) = \frac{1}{z},$$

or just Mathematica’s function `Series`, it can be shown that up to linear terms in $\kappa \delta$, $u$ and $v$ are

$$v \approx \beta + \delta, \quad w \approx \beta + (1 + \kappa^2) \beta^2 \delta,$$

yielding

$$\sigma' \approx \frac{1}{d} \frac{[1 + (1 + \kappa^2) \beta \delta] \delta + [2 + (1 + \kappa^2) \beta \delta] \beta}{\delta + 2 \beta}, \quad \tau' \approx \frac{1}{d} \frac{\delta + [2 + (1 + \kappa^2) \beta \delta] \beta}{\delta + 2 \beta}.$$
As long as

\[ \left| 1 + \kappa^2 \right| \beta \delta \ll 1, \tag{40} \]

both \( \sigma' \) and \( \tau' \) simplify to \( 1/d \), just as \( \sigma \) and \( \tau \) (see Eq. (23)). Certainly, this holds for sufficiently small \( \beta \delta \) (i.e. \( \mu_\mathrm{r1} >> \delta \)), what includes both magnetic and non-magnetic shields in static as well as low-frequency magnetic fields. Concluding, Eqs. (21)-(22), and consequently, the approximate BEM model (24), should work properly for any values of \( |\kappa| \). Although the considerations concern a cylindrical shield, it seems to be in force also for shields of other shapes. This is confirmed in the subsequent numerical simulations.

Both models, the standard and approximate, were tested for various values of parameters \( \delta, \kappa, \) and \( \beta \), with quadratic or constant boundary elements. Results of numerical computations for exemplary values of parameters are shown in plots of boundary values of \( A \) and \( B_t = -\partial_n A^{(1)} \) (with normal direction outwards the shield). Values of \( A \) are given in units of \( B_0 R_1 \), values of \( B_t \) in units of \( B_0 \) and the horizontal axis identifies the index of boundary node. In some cases, plots of errors of potential \( (\delta A) \) and tangential component of magnetic field intensity \( (\delta B_t) \) in particular boundary nodes are more informative. The errors are defined as follows:

\[ \delta A = \frac{A_{\text{num}} - A_{\text{th}}}{|A_{\text{th}}|_{\text{max}}} \cdot 100\%, \]  

\[ \delta B_t = \frac{B_{\text{num}} - B_{\text{th}}}{|B_{\text{th}}|_{\text{max}}} \cdot 100\%, \]  

where “th” and “num” refer to the theoretical value and its numerical estimate, respectively. Quantities \( |A_{\text{th}}|_{\text{max}} \) and \( |B_{\text{th}}|_{\text{max}} \) are the maximal values of \( |A_{\text{th}}| \) and \( |B_{\text{th}}| \) on boundary \( S_1 \) or \( S_2 \).

Figure 3 shows values of \( |A| \) and \( |B| \) for \( \delta = 0.1, \ k_\mathrm{c} = 10, \mu_\mathrm{r1} = 1 \) and 1000, and constant boundary elements (for quadratic elements the results are very similar). This cases correspond to a relatively thick non-ferromagnetic or ferromagnetic EM shield. The approximate model gives quite accurate results, although the standard BEM is more accurate in this case (due to large enough \( \delta \)).

Figure 4 show values of \( |\delta A| \) and \( |\delta B_t| \) for \( \delta = 0.01, \ k_\mathrm{c} = 10, \mu_\mathrm{r1} = 1 \) or 1000 for constant boundary elements, and Figure 5 – for quadratic elements. In both cases, the approximate BEM model gives errors comparable with the standard BEM model, and they usually are below 1%. Surprisingly, quadratic elements give unpleasant oscillations in boundary values, and errors \( |\delta B_t| \) are larger for the standard BEM.

Figure 6 show values of errors \( |\delta A| \) and \( |\delta B_t| \) for the same parameters, except for \( k_\mathrm{c} = 100 \), and constant elements. This time the approximate model gives more accurate results, with errors below 1%, whereas the standard BEM leads to considerable errors.
Fig. 3. Magnitudes of potential $A$ (a, c) and tangential component of magnetic flux density $B_t$ (b, d) on boundaries $S_1$ and $S_2$ for $\delta = 0.1$, $k = 10$ with $\mu_1 = 1$ (a, b) or $\mu_1 = 1000$ (c, d)

Rys. 3. Magnitudy potencjału $A$ (a, c) i składowej stycznej indukcji magnetycznej $B_t$ (b, d) na brzegach $S_1$ i $S_2$ dla $\delta = 0.1$, $k = 10$ oraz $\mu_1 = 1$ (a, b) i $\mu_1 = 1000$ (c, d)

Fig. 4. Magnitudes of errors $\delta A$ (a, c) and $\delta B_t$ (b, d) on boundaries $S_1$ and $S_2$ for $\delta = 0.01$, $k = 10$ with $\mu_1 = 1$ (a, b) or $\mu_1 = 1000$ (c, d) for 32 constant elements

Rys. 4. Magnitudy błędów $\delta A$ (a, c) i $\delta B_t$ (b, d) na brzegach $S_1$ i $S_2$ dla $\delta = 0.01$, $k = 10$ oraz $\mu_1 = 1$ (a, b) i $\mu_1 = 1000$ (c, d) dla 32 elementów stałych
Fig. 5. Magnitudes of errors $\delta A$ (a, c) and $\delta B_t$ (b, d) on boundaries $S_1$ and $S_2$ for $\delta = 0.01$, $k = 10$ with $\mu_{r1} = 1$ (a, b) or $\mu_{r1} = 1000$ (c, d) for 16 quadratic elements

Rys. 5. Magnitude błędów $\delta A$ (a, c) i $\delta B_t$ (b, d) na brzegach $S_1$ i $S_2$ dla $\delta = 0.01$, $k = 10$ oraz $\mu_{r1} = 1$ (a, b) i $\mu_{r1} = 1000$ (c, d) dla 16 elementów kwadratowych

Fig. 6. Magnitudes of errors $\delta A$ (a, c) and $\delta B_t$ (b, d) on boundaries $S_1$ and $S_2$ for $\delta = 0.01$, $k = 100$ with $\mu_{r1} = 1$ (a, b) or $\mu_{r1} = 1000$ (c, d) for 32 constant elements

Rys. 6. Magnitude błędów $\delta A$ (a, c) i $\delta B_t$ (b, d) na brzegach $S_1$ i $S_2$ dla $\delta = 0.01$, $k = 100$ oraz $\mu_{r1} = 1$ (a, b) i $\mu_{r1} = 1000$ (c, d) dla 32 elementów stałych
Figure 7 concerns a thin ($\delta = 0.01$) magnetic shield ($\mu r_1 = 1000$) with $\hat{k} = 0$, what corresponds to a static magnetic field ($\omega = 0$), or non-conductive shield ($\gamma_1 = 0$). Both models give results of comparable errors. Since $\delta \mu r_1 = 10^{-5} << 1$, the approximate model works well also in this case.

3.3. C-shape shell

To make sure the approximate model works good not only for shields of circular cross-section, a more complicated shape was also tested. This was a C-shape shell of internal and external radii $a$ and $2a$, respectively, and gap angle of $90^\circ$. The shape is visible in small insets in Figure 8. The relative thickness was defined as $\delta = d/a$, and the dimensionless parameter of skin effect $\hat{k} = \hat{k} + j\hat{k} = \kappa a$. Figures 8-9 show magnitudes of potential $A$ and tangential component of magnetic flux density $B_t$ for several sets of parameter values.
Both models give similar results, although there are small differences (Figures 9a and d). To find out which model is more accurate, additional tests involving comparisons with results given by other methods are required. However, the differences are not large, so that even if the approximate model is less accurate in these cases, its evident advantages (smaller system of equations, no nearly singular integrals) should partially compensate its possible lacks.

4. CONCLUSIONS

The approximate BEM model, which combines BEM and semi-analytical solution, was proposed as a method of analyzing the time-harmonic magnetic field nearby thin closed shells, like long electromagnetic shields of constant cross-section, e.g. cylindrical. Small thickness of such shells can be a serious problem in numerical computations. The standard BEM model works well for sufficiently thick shells. For thinner shells it requires very accurate evaluating of nearly singular integrals. Numerical tests showed that this can be unreachable even with use of very sophisticated methods of numerical integration, like those which have been built in the Mathematica. As a result, it can give considerable errors if the
thickness of the shell is too small. On the other hand, the approximate BEM model works the better the smaller \( \delta \), and no nearly singular integrals need evaluation. Both theoretical analysis of cylindrical shell of circular cross-section and numerical tests for various shaped shells show that the approximate BEM model works well for large values of \(|\kappa|\), and also for smaller \(|\kappa|\) with additional requirement that \( \delta/\mu_1 \) should be sufficiently small. The conditions are usually fulfilled in possible practical applications. It is interesting that constant elements usually give smaller numerical errors than quadratic do (with the same set of field approximation nodes and the same order of Gaussian quadratures).

The key advantages of the approximate BEM in comparison with the standard BEM, are:
- smaller equation system,
- no nearly singular integrals (for sufficiently regular boundary),
- no need to evaluate BEM integrals with fundamental solution (10),
- good for very thin shells.

The main disadvantage is that it uses an approximate solution for the layer, which cannot be proved to be correct for all cases. Despite this, the approximate model is worth taking into account. Its usability in the considered class of problems was confirmed. Further research should focus on:
- introducing varying thickness,
- developing the model for open shells.

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